Groundwater-Surface water Modeling: A Critical Review

Shikha Saxena[1], Dr. Rajeev K Bansal[2], Dr. Basant Singh[3] ,Dr Chhaya k Lande[4]
[1]Reasearch Scholar, Department of Mathematics, RNTU, Bhopal (M.P.) India.
[2]Associate Professor, National Defence Institute, Khadakvasala,Pune (M.S.) India.
[3] Pro Vice Chancellor, Dr. C. V. Raman University, Patna (Bihar) India.
[4]Assistant Professor, Department of Mathematics, RMDSSOE, Pune (M.S.) India.

ABSTRACT
Surface water-Groundwater interaction is a complex process. Mathematical modelling is the emerging technique to understand the water head variation in the aquifer, flow of groundwater, recharge effect. This paper gives the critical review of this process. Many researchers developed mathematical model which simulates the flow of groundwater under different hydrological conditions. In this papers step by step overview of these models is presented. In addition to this the effect of research and discharge processes is also comprehensibly outlined. The different methods for solving the partial differential equation are also pointed out. This article provides the overview of almost all mathematical models developed in this era. This overview concludes with a discussion of research needs and challenges facing this evolving field.

Keyword: Business Equation, Laplace Transform, Fourier Transform, Sloping Aquifer

I INTRODUCTION

Surface water groundwater are not separate entities but are the component of hydrological cycle. Groundwater bodies are often connected with surface water bodies and affect each other quantity and quality wise. In the past few decades, increased demand of water due to industrialization, population growth and excess withdrawal, has enable to think about the proper environmental management and aquifer recharge. Surface groundwater interaction plays an important role in catchment hydrology and base flow analysis. Hence prior knowledge of aquifer response to recharge and withdrawal is very important for planning and implementing the resource related project.

Mathematical modelling is the emerging tool for predicting water table behaviour in many complex geological situations. Many models were developed for water table fluctuation in aquifer but on restrictive assumptions that the aquifer base is horizontal (Moench and Barlow 2000). In a real-lifeaquifer base are often sloping and connected to each other with the leaky formation. Some of the publication adhere this topic of water table fluctuation in sloping aquifers but they are few. Moreover, effect of recharge and withdrawal on water table variation is not thoroughly studied by the researchers. It is important for any water managers all researchers to understand all facts of surface water groundwater mechanism.

The aim of this paper is to give systematic and critical review of the models developed for water table fluctuation when aquifer is lying on sloping base. The focus is on the estimation of water fluxes at the stream aquifer interaction. It is intended to provide overview of mathematical equations and boundary conditions used in the model based on sloping aquifers. The mathematical equation which governed the water flow in horizontal and sloping aquifers are discussed.

II MATHEMATICAL FORMULATION OF GROUND WATER FLOW

Groundwater always moves from an area of higher hydraulic gradient to anarea of lower hydraulic gradient. Hydraulic head is the driving force which helps groundwater to flow. Hydraulic gradient is approximately the slopes of water table in a simple unconfined water system. Water table contour lines essentially represent the water table elevation which is called hydraulic head as mentioned above. Water table contour lines, also known as equipotential lines, are used to determine the direction of water flow in the region.

Surface water is connected with ground in almost all types of geological formations. Hence the surface water bodies are part of groundwater flow system. Though the surface water is segregated from groundwater by an unsaturated region, percolation of surface water affects the quantity of groundwater. The exchange of these two water bodies through unsaturated zone affects both quantity and quality wise. The geological formations of rocks and aquifer are the major components affect the flow of surface-groundwater movement. Along this climate change, effects of precipitation and evaporation, removal of water, distribution of water are some of the important issue take in consideration while studying groundwater hydrology and hydro system.

The seepage flow in saturated subsurface zones is primarily governed by the principle of local hydraulic gradient, known as Darcy's law (Darcy, 1866). The law state that the rate at which the fluid flow in the permeable medium is directly proportional to the drop-in elevation, the cross -sectional area and inversely proportional to the distance between them.

Therefore, the total discharge 'Q' is given by the equation

$$Q = -KA\frac{\partial(h_1 - h_2)}{\partial l} \quad (1)$$

where K is proportionality constant, called hydraulic conductivity.
When the aquifer is isotropic, the hydraulic conductivity is same in all direction. So Darcy's law can be expressed as,

$$q = -K\nabla h \quad (2)$$

where q is the Darcy's velocity vector, K is hydraulic conductivity and∇h is the gradient of hydraulic head.The basic equation of groundwater flow can be derived by applying the principle of mass conservation on a Representative Elementary Volume (REV).The conservation of mass states that for a given increment of time, difference between the mass flowing across the boundaries, the mass flowing out across the boundaries and the sources within the volume is the change in storage. In other words

Mass inflow rate – Mass out flow rate= Change of mass storage with time
The equation thus obtain is

$$-\nabla\cdot(\rho\, q) = \frac{\partial}{\partial t}(n\rho) \quad (3)$$

In rectangular Cartesian coordinates, the flow equation is given by

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right) = S\frac{\partial h}{\partial t} \quad (4)$$

Equation (4) is main equation of ground water flow in saturated media.
K_x, K_y and K_z are hydraulic conductivitiesalong x–, y– and z–axes for anisotropic medium. It can be written in many forms that to apply different conditions, such as

a) Under steady-state-flow condition ($\partial h/\partial t = 0$) for anisotropic medium

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right) = 0 \quad (5)$$

If the porous medium is isotropic (K_x=K_y = K_z) and homogeneous then the equation (5) is

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (6)$$

In this equation the hydraulic head is not transient and steady state condition is prevailing. Equation (6) is the known as the Laplace equation.

For isotropic and homogeneous medium, equation of groundwater flow can be written as

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial h}{\partial z}\right) = \frac{S}{K}\frac{\partial h}{\partial t} \quad (7)$$

1) The flow equation with thickness of aquifer is 'b' is given by

$$\frac{\partial}{\partial x}\left(T_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(T_z\frac{\partial h}{\partial z}\right) = S\frac{\partial h}{\partial t} \quad (8)$$

Where T_x=K_x.b, T_y=K_y.b,T_z=K_z.b

2) If the hydraulic head is constant in vertical direction, then the equation of flow in two dimensions is

$$\frac{\partial}{\partial x}\left(T_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y\frac{\partial h}{\partial y}\right) = S\frac{\partial h}{\partial t} \quad (9)$$

III GROUND WATER FLOW EQUATION IN SLOPING AQUIFER

The nonlinear Boussinesq equation obtained by combining the Darcy's law with mass conservation principle has been the governing equation in many researches pertaining to subsurface seepage flow. Polubarinova-Kochina (1962), Jacob Bear (1972) presented an extensive analysis of subsurface seepage flow, addressing variety of fundamental issues of surface-groundwater interactions. In their research different modelling techniques is used along with the analytical and numerical solution under different boundary conditions. Wooding (1966) used the hodograph method to obtain the exact solution for groundwater flow over a sloping impermeable layer, the approximate solution so found using extended Dupuit -Forchheimer is in full agreement with exact results.

The landmark development in the contemporary literature was in 1971 when Child (1971) developed the mathematical approximation of groundwater flow over sloping impervious bed by using extended Dupuit-Forchheimer assumptions. Using conformal mapping, Wooding (1966) presented analytical solution of saturated seepage flow over sloping base. Later, several researchers used this approximation to analyse groundwater flow system under various conditions. A simplification of Child (1971) approximation was presented by Chapman (1980) who used a horizontal and vertical axes system to describe the groundwater flow. Along with Child Hunt (1971) presented an analytical solution for water table fluctuation in response to recharge from circular basin.

Chapman (1980) developed the above equation by considering assumptions as i) For small inclination of free surface stream lines are considering nearly horizontal ii) Hydraulic gradient is equal to slope of free surface and does not vary with the depth. The equation is as follows

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right)-\tan\theta_x\frac{\partial h}{\partial x}-\tan\theta_y\frac{\partial h}{\partial y}+\frac{Q}{K}=\frac{S}{K}\frac{\partial h}{\partial t} \qquad (10)$$

The above equation is second order non-linear differential equation, parabolic in nature often referred as Boussinesq equation. This equation is used when the subsurface drainage is over moderately sloping beds. If tan θ_x and tan θ_y are the downward bed slopes along the positive directions of x and y-axes, respectively.

Rao and Sharma (1981) have presented an analytical solution to describe water table fluctuation in finite aquifer system in response torecharge from a rectangular basin.Raiet al.(1994), Rai and Singh (1995) took an effort to developanalytical solutions by considering time dependent recharge rate.In these studies, the recharge rate is approximatedby simple functions such as periodic and exponential functions.Manglik et al (1996) developed an analytical solution for water table variation in an unconfined aquifer induced by time varying recharge from a rectangular basin. Finite Fourier transform method is used to find the solution.

Koussiset al (1998) studied the subsurface drainage flow in a soil layer which is resting on sloping bed. The linearized one-dimensional Boussinesq equation is used as governing equation which is extended to allow the leakage through underlying base. Seepage on hill slope problem is further studied by Shukla et al (1999). Intheir study Boussinesq equation is solved for the drainage of sloping landsresting on impermeable layer.

Investigation related to hydraulic head and flow rate in a slopping aquifer, whenthe aquifer is in nearby contact bodies, have been done by many researchers such asPinder and Sauer (1971), Zlotnik and Huang (1999), Verhoest and Troach (2000),Upadhyaya and Chauhan (2000,2002). Tartakovsky (2008) developed new model forfalling water table between two drains laying on a horizontal and sloping impermeablebarrier using finite difference scheme. Butler and Zlotnik (2001) presenteda new solution for estimation of drawdown and stream depletion produced by pumping. Parlange et al (2001) suggested improvement in the solution of Boussinesqequation for a sudden drawdown in a saturated horizontal aquifer. Kim and Joon (2001)analyzed variation of a water table in a horizontal unconfined aquifer receiving uniform recharge mathematically by solving Boussinesq equation as a governing equation. Inclined ditch-drained aquifer by considering the temporally verifying recharge rateis studied by Valentijn et al (2002) and introduced a solution of Boussinesq equation. The unique boundary condition like a steady state constant recharge rate is used as aninitial condition. The solution is used for assessing the influence of a variety of hydrogeological factors on drawdown and stream depletion calculation and demonstrates the magnitude of error introduced by commonly used analytical methods.

IV ANALYTICAL MODELS

Many of the researchers used analytical method to solve groundwater flow equation.

Most of the governing equations are in the form of partial differential equations and can be solved analytically. The commonly used methods to solve PDE analytically are Laplace transform, Fourier Transforms, Melin Transforms etc. In case of nonlinear partial differential equation which does not admit the solution, Linearization is used.

It is worth mentioning that the work carried out by Aklyas and Koussis (2007) provides fundamental modelling tools for estimating surface-groundwaterinteraction induced by instantaneous stream-stage variation in the presence of bedslope and vertical sedimentary layer; however, the study does not shed any light onthe flow mechanism when the rise and decline in the stream head is gradual. Indeed, the variations in stream stage because of rainfall and snow melt consist of a risinglimb, peak point and recession limb which can be approximated by exponential decaying function (Teloglou and Bansal 2012; Bansal and Teloglou 2013). Simulationof stream stage using a sequence of piecewise linear segments of varying lengths andslopes is also possible. This technique was proposed by Manglik et al. (1997) for approximation of vertical recharge and subsequently used by Rai et al. (2006), Rai and Manglik (2012) and Manglik et al. (2013). However, use of exponential function in analytical modelling is more suitable for its mathematical simplicity and low computational cost. Mathematical modeling of surface Groundwater interaction over sloping Terrain. Lande et al (2013) used Laplace Transform technique to show the variation of water head in the aquifer, flow rate at the stream aquifer interface which is in contact with a constant Piezometric level at one end and a stream of time varying water level at other end. Later Bansal et al (2016) shows the Simulation of 2-d subsurface seepage flow in an anisotropic porous medium. In this study a new analytical solution to estimate the water table fluctuation in an anisotropic unconfined aquifer is developed. Parabolic non- linear Partial Differential Equation is solved using Fourier Transform Technique. The solution obtained in the study has ability to predict the recharged effect and it is seen in the form of water mound beneath the recharge basins and the cone of depression under the wells. Zlotnikandet al (2017) present an analytical expression for a transient and steady state groundwater mound in a sloping aquifer under MAR (Managed Aquifer Recharge) This studies consider general orientation of the recharge basins at arbitrary angle with x –axis .A technique is used developed for the orientation of the recharge source. Dupit-Forchheimer approximation and green's function method are used for finding the mound height. The solution reveals the effect of dip angle on the elevation of mound. These findings help in determining proper rate of recharge and various MAR scenarios. Bhandari et al (2018)approximate the analytical solution obtained forstream–aquifer interaction assuming one-dimensional horizontal ground water flow in homogeneous and isotropic aquifer. Under the assumption of homogeneous and isotropic aquifer, the nonlinear Boussinesq equation with recharge is solved usingvariational homotopy perturbation method.

V NUMERICAL MODELS

One of the strong approaches of solving PDE are Numerical methods. Some commonly used numerical methods in the literature are Predictor corrector Method, Finite difference method, Finite element Method, Adomain decomposition, Perturbation method.

Kalaidzidou et al (1996) presented numerical solution using perturbation method. Jianping et al (2011) introduced a new analyticalsolution of linearized Boussinesq equation which describes groundwater table variationin a semi-infinite unconfined aquifer when the adjacent reservoir water drops downwith a constant speed. The solution is verified by comparing it with a numberof numerical solutions of the nonlinear Boussinesq equation.

Antangana and Botha (2012) used the homotopy decomposition method to solvethe groundwaterflow equations. Bansal et al(2013)used a fully explicit Predictor Corrector method to validate the efficiency of linearization. The results obtained shows the considerable changes in aquifer's water head profile mainly depends on the bed slopes, riserate of the stream level and the recharge rate. Nania et al(2014) described the application of model for simulating dual drainage in urban areas.This model consists of four models which simulates rainfall runoff transformation, new evacuation by the inlets located in the streets, one dimensional flow routing onstreet network, flow interaction between surface water on the streets and undergroundstream water system.Later in 2015 Jiangand Tang (2015) proposed a general approximate method to predict the aquiferresponse subject to water level variations in a free water body. Theproposed methoddecomposes the nonlinear PDE in to linear diffusion equation and two nonlinearcorrection functions. Solution is derived using general analytical method as well asadaptive - finite volume method. The analytical solution is applied to four differentsituations of water level i.e. constant, sudden rise/fall, linear and periodic change.Shaikh et al(2018)analyze the dynamic behaviour of tide induced water table variations in an unconfined aquifer system. Nonlinear Boussinesq equation is solved analytically to examine the effect of tidal oscillations on water table height. These equations are further linearized with a dynamic iterative scheme. The results can be used for validation of numerical models and to understand complicated groundwater tidal wave interaction in complex hydro geographical coastal beaches.

VI CONCLUSION

Analytical modelling of subsurface seepage is an important aspect in groundwater hydrology as it provides useful insight into the groundwater flow mechanism. Mathematical models describing surface-groundwater interaction are gaining popularity due to their cost effectiveness and ability to handle varying hydrological conditions. In this review paper, we have started with the flow characteristic in horizontal aquifer. The study carried out by various researchers for horizontal aquifer is briefly presented. Furthermore, the flow equation on sloping aquifer is described and the essential tools to solve this equation are adequately discussed. A comprehensive survey of the relevant mathematical models for surface-groundwater interaction over sloping beds is presented in a chronological order.

REFERENCES

[1] R.A.Wooding, "Ground water flow over a sloping impermeable layer ,2.exact solution of conformal mapping" Journal of Geophysical Research, Vol 7, no12,1966

[2] E.C.Chlids,"Drainage of Ground water resting on a sloping bed",vol 7, no 5,Water Resource Research, 1971

[3] T.G.Champman," Modelling ground water flow over sloping bed" Water Resources Research16, no 6,1980

[4] P.S. Huyakorn,B.H.lester,C.R.Faust,"Finite element techniques for modelling ground water flow in fractured aquifer" Water Resources Research, Vol-19, No-6,1983

[5] Latinopoulous-P," Periodic recharge to finite aquifer from rectangular areas" Adv water resources, Vol 7,1984

[6] A.D. Koussis, "A Linear Subsurface storm flow model", Water Resources Research, vol-28, no-4,1992

[7] KarmouzisD.N., Tloglou I., "Analytical solution of unsteady drainage problem for soil subjected to variable recharge from a semi unconfined aquifer"Water resources management-7,1993

[8] A. Manglik, S.N.Rai,R.N.Singh," Response of an unconfined aquifer induced by time varying recharge from rectangular basin" Water resources Management-11,1997

[9] A.D.Koussis,M.E.Smith,E.Akylas,M.Tombrou, "Ground water Drainage flow in a soil layer resting on an inclined leaky bed" Water resources research,Vol-34,N0-11,1998

[10] W.L.Hogarath, J.Y.Parlange, M.B.Palange, D. Lockington, "Approximation analytical solution of the Boussinesq equation with numerical validation" Water Resources Research, Vol-35,No-10,1999

[11] B. Hunt, "Unsteady stream depletion by ground water pumping" Ground Water,vol-37,no-1,1999

[12] A.F.Moench and P.M.Barlow, "Aquifer response to stream stage and recharge variation. I.analytical step response functions" ,Journal Of Hydrology 230, 2000

[13] A.F.Moench, P.M.Barlow, L.A.Desimone, "Aquifer response to stream stage and recharge variation. II.Convolutionmethod and applications", Journal of Hydrology 230,2000

[14] J.J.Butler,V.A.Zlotnik,M.S.Tsou, "Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream" Earth and atmospheric sciences, 2001.

[15] A. Upadhaya,Chauhan.H.S., "Filling water table in horizontal/sloping aquifer" Journal of Irrigation and general engineering" 2001

[16] Y.C.Chang,H.D.Yeh, "Analytical solution for groundwater flow in an anisotropic sloping aquifer with arbitrarily located multiwalls" Journal of Hydrology-347,2007

[17] Akylas, E., A.D.Koussis, "Response of sloping unconfined aquifer to stage changes in adjacent stream. I.Threotical analysis and derivation system functions", Journal of Hydrology-338, 2007

[18] A.D.Koussis, Akylas.E., Mazi.K, "Response of sloping unconfined aquifer to stage changes in adjacent stream II. Applications" Journal of Hydrology-338,2007

[19] R.K. Bansal, Das.S.K. "Effect of bed slope on water head and flow rate at the interfaces between the stream and ground water: Analytical study" Journal of Hydrology -2007

[20] V.A.zlotnik, D.M.Tartavoski, "Stream depletion by groundwater pumping in leaky aquifer"Journal of Hydrologic Engineering,2008

[21] Chung.M.S., Huang.C.S.,G.H. Li,H.D. Yeh, "Ground water Fluctuation in heterogeneous costal leaky aquifer system" Hydrology and earth system sciences discussion,2010

[22] K.N. Moutopolous, "The Analytical solution of the Boussinesq equation for induced by a step change of the Water Table Elevation revisited" Trans porous med.-85,2010

[23] A.Atangana, and J.F.Botha, "Analytical solution for ground water flow equation obtained via Homotopy Decomposition method.", Earth sciences and Climate change, vol-3, no-2, 2012

[24] I.S. Teloglou, R.K.Bansal, "Transient solution for stream-unconfined aquifer interaction due to time varying stream head and in the presence of leakage" Journal of Hydrology-428-2012

[25] R. K. Bansal, C.K.Lande, A. Warke, "Unsteady groundwater flow over sloping beds: analytical quantification of stream–aquifer interaction in presence of thin vertical clogging layer" Journal of Hydrologic Engineering, Vol 21/7, pp04016017-1 to 04016017-2015

[26] V.A.zlotnik, A Kacimov, "Estimating groundwater mounding in sloping aquifers for Managed Aquifer Recharge"National Ground Water Association,2017

[27] B.Y.Shaikh, R.K.Bansal, S.K.Dass, "Propagation of Tidal wave in coastal Terrains with Complex Bed Geometry" Springer -Environmental processesVol-5,Issue-3pp 519-537 ,2018

[28] A.D.Bhandari,V.H.Pradhan,R.K.Bansal, "New analytical solution for stream -aquifer interaction under constant replenishment" Applied water science ,8: 171 ,2018