WAVELET TRANFORMATIONS & ITS APPLICATIONS IN DIGITAL IMAGE PROCESSING

A.H.M. Jaffar Iqbal Barbhuiy
Department of Comp Science
Assam University
Silchar, Assam, India
jaffar.iqbal84@gmail.com

Manimala Singha
Department of Comp Science
Assam University
Silchar, Assam, India
n.manimala888@gmail.com

K. Hemachandran
Department of Comp. Science
Assam University
Silchar, Assam, India
khchandran@rediffmail.com

Abstract

Image processing based on the continuous or discrete image transforms are classic techniques. The image transforms are widely used in image filtering, data description, etc. Considering that the Haar and Morlet functions are the simplest wavelets, these forms are used in many methods of discrete image transforms and processing. Wavelets transforms are widely used in many research areas and its advantages over conventional Fourier transform as it takes less time and individual wavelet functions are localized in space but Fourier transform cannot. In this study, similarities and dissimilarities between wavelet transform and Fourier transform are also discussed. Furthermore, the applications of wavelet transform in many areas like face recognition, fingerprint analysis, image compression and image denoising are discussed.

Keywords:- wavelet transform, image filtering, data description.

I. INTRODUCTION

The word "wavelet" has been introduced by Morlet and Grossmann [1] in the early 1980s. They used the French word ondelette, meaning "small wave" originated from the study of time-frequency signal analysis, wave propagation, and sampling theory. Morlet first introduce the idea of wavelets as a family of functions constructed by using translation and dilation of single function, called mother wavelets, for analysis of nonstationary signals. Wavelets are a mathematical tool that can be used to extract information from many different kinds of data, including audio signals and images. The subject of wavelet analysis has recently drawn a great deal of attention from mathematical scientists in various disciplines. It is creating a common link between mathematicians, physicists, and electrical engineers with modern application as diverse as wave propagation, data compression, image processing, pattern recognition, computer graphics and other medical image technology. Sets of wavelets are generally needed to analyze data fully. The wavelet transform decompose the signal with finite energy in the spatial domain into a set of function as a standard in the modular spatial domain of orthogonal. Then we analyze the characteristics of the signal in the modular spatial domain. Compared with the traditional Fourier analysis, the wavelet transform can analyze the function in the modular spatial domain and timing domain which has better local capacity of the frequency and time. It is the development and sublimation of Fourier transform, which has a lot of advantages.

The main objective of wavelet transform is to define the powerful wavelet basis functions and find efficient methods for their computation. Fourier methods are not always good tools to recapture the signal or image, particularly if it is highly non–smooth. Too much Fourier information is needed to reconstruct the signal or image locally. The wavelet analysis is done similar to the Short Time Fourier Transform (STFT) analysis. The signal to be analyzed is multiplied with a wavelet function just as it is multiplied with a window function in STFT, and then the transform is computed for each segment generated. However, unlike STFT, in Wavelet Transform, the width of the wavelet function changes with each spectral component. The Wavelet Transform, at high frequencies, gives good time resolution and poor frequency resolution, while at low frequencies, the Wavelet Transform gives good frequency resolution and poor time resolution. In these cases the wavelet analysis is often very effective because it provides a simple approach for dealing with the local aspects of a signal, therefore particular properties of the Haar wavelet transforms allow to analyze the original image on spectral domain effectively. Wavelet transforms have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology.

The first literature that relates to the wavelet transform is Haar wavelet. It was proposed by the mathematician Alfred Haar in 1909[2, 3]. However, the concept of the wavelet did not exist at that time. Until 1981, the concept was proposed by the geophysicist Jean Morlet [1-10]. Later, Morlet and Grossman invented the term wavelet in 1984. Before 1985, Haar wavelet was the only orthogonal wavelet known to the people[2-3]. Fortunately, the Mathematician Yves Meyer constructed the second orthogonal wavelet called Meyer wavelet in 1985 [4-7]. As more and more scholars joined in this field, the 1st international conference was held in France in 1987[8].

In 1988, Stephane Mallat and Meyer proposed the concept of multiresolution [9]. In the same year, Ingrid Daubechies found a systematical method to construct the compact support orthogonal wavelet. In 1989, Mallat proposed the fast wavelet transform. With the appearance of this fast algorithm, the wavelet transform had numerous applications in the signal processing field [10].

Summarize the history. We have the following table:

1910, Haar families, which was proposed by the mathematician Alfrd Haar. Haar wavelet is the first literature relates to the wavelet transform, but the concept of the wavelet did not exist at that time[2-3].

1981, Morlet, wavelet concept, which was proposed by the geophysicist Jean Morlet [1-8].

1984, Morlet and Grossman, "wavelet". Morlet and the physicist Alex Grossman invented the term "wavelet"[1].

1985, Meyer, "orthogonal wavelet". Before 1985, a lot of researchers thought that there was no orthogonal wavelet except Haar wavelet. The mathematician Yves Meyer constructed the second orthogonal wavelet called Meyer wavelet in 1985[1-10].

1987, International conference in France, which is the 1st international conference about Wavelet transform [1-10].

1988, Mallat and Meyer, multiresolution. Stephane Mallat and Meyer proposed the concept of multiresolution[1-10].

1988, Daubechies, compact support orthogonal wavelet. Ingrid Daubechies found a systematical method to construct the compact support orthogonal wavelet [3].

1989, Mallat, fast wavelet transform. With the appearance of this fast algorithm, the wavelet transform had numerous applications in the signal processing field [10].

II. BASIC DEFINITIONS AND AN OVERVIEW OF WAVELET TRANSFORMS

A wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components. Usually one can assign a frequency range to each scale component. Each scale component can then be studied with a resolution that matches its scale. A wavelet transform is the representation of a function by wavelets. The wavelets are scaled and translated copies (known as "daughter wavelets") of a finite-length or fast-decaying oscillating waveform (known as the "mother wavelet").

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right),\quad a, b \in \mathbb{R}, a \neq 0, \qquad (1)$$

Where ψ is a wavelet function, a, is a scaling parameter which measure the degree of compression or scale, and b, is a translation parameter which determines the time location of the wavelet.

Wavelet transforms have advantages over traditional Fourier transforms for representing functions that have discontinuities and sharp peaks, and for accurately deconstructing and reconstructing finite, non-periodic and/or non-stationary signals.

A WAVELET VS FOURIER TRANSFORMS

Similarities between Fourier and Wavelet Transforms, the Fast Fourier transform (FFT) and the discrete wavelet transform (DWT) are both linear operations that generate a data structure that contains segments of various lengths. The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FFT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain. For the FFT, this new domain contains basis functions that are sines and cosines. For the wavelet transform, this new domain contains more complicated basis functions called wavelets. Both transforms have another similarity. The basis functions are localized in frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and scalegrams useful at picking out frequencies and calculating power distributions.

The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space. Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes

many functions and operators using wavelets "sparse" when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series.

III. DISCRETE WAVELETS TRANFORMATIONS

The Wavelet Series is just a sampled version of Continuous Wavelet Transform (CWT) and its computation may consume significant amount of time and resources, depending on the resolution required. If the function being expanded is a sequence of numbers, like samples of a continuous function f(x), the resulting coefficients are called the Discrete Wavelet Transform (DWT) of f(x).In this case, the series expansion of wavelet transform in one dimension is given below.

$$W_\varphi(j_0, k) = \frac{1}{\sqrt{M}} \sum_x f(x) \varphi_{j_0,k}(x) \qquad (2)$$

$$W_\psi(j, k) = \frac{1}{\sqrt{M}} \sum_x f(x) \psi_{j,k}(x) \qquad (3)$$

for $j \geq j_0$ and

$$f(x) = \frac{1}{\sqrt{M}} \sum_k W_\varphi(j_0, k) \varphi_{j_0,k}(x) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_k W_\psi(j,k) \psi_{j,k}(x)$$

Here, $f(x)$, $\varphi_{j_0,k}(x)$, and $\psi_{j,k}(x)$ are functions of the discrete variable x=0,1,2……M-1.

For Haar wavelets, the discretized scaling and wavelet functions employed in the transform (called the basis functions)correspond to the rows of M×M Haar transformation matrix. The transform is composed of M coeffiecients, the minimum scale is 0,and the maximum scale is J-1.The coefficients defined in equation (2) and (3) are usually called approximation and detail coefficients respectively [11].

The discrete wavelet transform in two dimensions of functions f(x,y) of size M×N is then

$$W_\varphi(j_0, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \varphi_{j_0,m,n}(x,y)$$

$$W_\psi^i(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \psi_{j,m,n}^i(x,y)$$

$i = \{H, V, D\}$.

Where in the one-dimensional case, j_0 is an arbitrary starting scale and the $W_\varphi(j_0, m, n)$ coeffients define an approximation of f(x,y) at scale j_0. $W_\psi^i(j, m, n)$ coefficients add horizontal, vertical, and diagonal details for scales $j \geq j_0$ and i is a superscript that assumes the values H,V, and D [11].

The Discrete Wavelet Transform (DWT), which is based on sub-band coding is found to yield a fast computation of Wavelet Transform. It is easy to implement and reduces the computation time and resources required In CWT, the signals are analyzed using a set of basis functions which relate to each other by simple scaling and translation. In the case of DWT, a time-scale representation of the digital signal is obtained using digital filtering techniques. The signal to be analyzed is passed through filters with different cutoff frequencies at different scales[11].

IV. APPLICATIONS OF WAVELET TRANSFORM IN IMAGE PROCESSING

Wavelets are a powerful statistical tool which can be used for a wide range of applications. Wavelet transforms are now being adopted for a vast number of applications, often replacing the conventional Fourier Transform. Wavelet Transforms (WT) can also be used in the field of Image compression, Feature extraction, image denosing and other medical image technology.

Many areas of physics have seen this paradigm shift, including molecular dynamics, astrophysics, density-matrix localization, seismic geophysics, optics, turbulence and quantum mechanics. This change has also occurred in image processing, blood-pressure, heart-rate and ECG analysis, DNA analysis, protein analysis, climatology, general signal processing, speech recognition, computer graphics and multifractal analysis. Some of the important applications of wavelet transform are described here.

A. FINGERPRINT RECOGNITION

Fingerprint verification is one of the most reliable personal identification methods and it plays a very important role in forensic and civilian applications. For automatic identification, it is one of the oldest and most reliable methods because of invariance of the fingerprint features over the age of the subject. Facsimile scans of the impressions are distributed among law enforcement agencies, but the digitization quality is often low. Because a number of jurisdictions are experimenting with digital storage of the prints, incompatibilities between data formats have recently become a problem. This problem led to a demand in the criminal justice community for a digitization and a compression standard, To overcome this problem. Federal Bureau of Investigation (FBI) has proposed a standrard for image compression using wavelets known as wavelet Scalar Quantization (WQS). This widely accepted in the industry as the defacto standard which has collected about 30 million sets of fingerprints [12, 13]. Other papers of wavelets transformation in fingerprint recognition are [14] and [15]. Shashi et al. [15] proposed a Discrete Wavelets Transformation based Fingerprint Recognition using Non Minutiae Features. The features of fingerprint such as Directional Information, Centre Area and Edge Parameters are extracted from DWT. Pokhriyal et al. [15] proposed an algorithm of fingerprint verification based on wavelets and pseudo Zernike moments. Wavelet was used to denoise and extract ridges.

B. IMAGE COMPRESSION

Image compression is one of the most important and successful applications of the wavelet transform. The rapid increase in the range and use of electronic imaging justifies attention for systematic design of an image compression system and for providing the image quality needed in different applications.

Image compression algorithms aim to remove redundancy in data in a way which makes image reconstruction possible." This basically means that image compression algorithms try to exploit redundancies in the data; they calculate which data needs to be kept in order to reconstruct the original image and therefore which data can be 'thrown away'. By removing the redundant data, the image can be represented in a smaller number of bits, and hence can be compressed. Grgic et al. [16] presented a comparative study of different wavelet-based image compression systems.

Discrete wavelet transform is adopted to be the transform coder in both JPEG2000 [17] and still image coding and MPEG-4 [18] still texture coding. JPEG2000 is the emerging next generation still image compression standard. JPEG2000 is to be delivered and agreed as a full ISO International Standard by the end of the year 2000. With the inherent features of Wavelet transform, it provides multi-resolution functionality and better compression performance at very low bit-rate compared with the DCT (Discrete Continuous Time)-based JPEG standard [19].

C. IMAGE DENOISING

An image is often corrupted by noise in its acquisition and transmission. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features[20].Wavelet transform provides us with one of the methods for image denoising. Wavelet transform, due to its excellent localization property, has rapidly become an indispensable signal and image processing tool for a variety of applications, including denoising and compression. Wavelet denoising attempts to remove the noise present in the signal while preserving the signal characteristics, regardless of its frequency content.

1) Wavelet Thresholding

Wavelet thresholding (first proposed by Donoho[21]) is a signal estimation technique that exploits the capabilities of wavelet transform for signal denoising.

It removes noise by killing coefficients that are insignificant relative to some threshold. Researchers have developed various techniques for choosing denoising parameters and so far there is no "best" universal threshold determination technique.

Yansun et al. [22] introduced an effective wavelet transform domain noise filtration technique. This filter preserves edges and removes noise. Noise is preferentially removed from the wavelet transform data at a given scale by comparing the data at that scale to the correlation of the data at that scale with those at larger scales. Features are identified and retained because they are strongly correlated across scale in the wavelet transform domain. Noise is identified and removed because it is poorly correlated across scale in the wavelet transform domain. Features remain relatively undistorted because they are very well localized in space in the wavelet transform domain; therefore, edges remain sharp after filtration.

D. FACE RECOGNITION

Face recognition in our life such as identification of person using Credit cards, Passport check, Criminal investigations etc. The human face is an important object in image and video databases, because it is a unique feature of human beings and is ubiquitous in photos, news videos, and video telephony. Face detection can be regarded as a more general case of face localization. In face localization, the task is to find the locations and sizes of a known number of faces (usually one). In face detection, one does not have this additional information. There are several application areas where

Automated face recognition is a relatively new concept. Developed in the 1960s, the first semi-automated system for face recognition required the administrator to locate features (such as eyes, ears, nose, and mouth) on the photographs before it calculated distances and ratios to a common reference point, which were then compared to reference data. In the 1970s, Goldstein, Harmon, and Lesk [23] used 21 specific subjective markers such as hair color and lip thickness to automate the recognition. Face recognition can be used for both verification and identification (open-set and closed-set). [24] Proposed face recognition using Linear Discriminant Analysis (LDA) with wavelet transformation. LDA is one of the principal techniques used in face recognition systems. LDA is well-known scheme for feature extraction and dimension reduction. LDA using wavelets transform approach that enhances performance as regards accuracy and time complexity.

V. CONCLUSION

In this review paper, an attempt has been made to study the wavelet transform and its applications in digital image processing. Some of the equations of discrete wavelet transform are described. The review was conducted to study the different suitable areas of wavelet transforms and its applications in digital image processing.

VI. REFERENCES

[1] A. Grossmann, and J. Morlet, Decomposition of Hardy functions into square integrable waveletsof constant shape, SIAM J. Anal. 15, pp. 723-736, 1984.

[2] Patrick J. Van Fleet,"Discrete Haar Wavelet Transforms",PREP - Wavelet Workshop, 2006.

[3] http://online.redwoods.cc.ca.us/instruct/darnold/laproj/Fall97/PMorton/imageComp3/.

[4] Y. Meyer, Wavelets: Algorithms and Applications, Society for Industrial and Applied Mathematics, Philadelphia, 1993, pp. 13-31, 101-105.

[5] James S. Walker. 1999. A Primer on Wavelets and Scientific Applications.

[6] Applying the Haar Wavelet Transform to Time Series Information

[7] Ripples in Mathematics: the Discrete Wavelet Transform by Arne Jense and Anders la Cour-Harbo, Springer, 2001

[8] Chun-Lin, Liu," A Tutorial of the Wavelet Transform" February 23, 2010.

[9] S.G. MALLAT, "A Theory for Multiresolution Signal Decomposition:The Wavelet Representation", IEEE Transactions On Pattern Analysis And Machine Intelligence. Vol. I , No. 7. July, 1989

[10] Stephane Mallat, "A Wavelet Tour of Signal Processing -The Sparse Way"

[11] R. C.Gonzalez, R. E. Woods, Digital Image Processing, 2nd edition, chapter 7,"Wavelets and Multiresolution Processing", Pages 372-386.

[12] Bradley, J., C. Brislawn, and T. Hopper. The FBI Wavelet/Scalar Quantization Standard for Grayscale Fingerprint Image Compression, Tech. Report LA-UR-93-1659, Los Alamos Nat'l Lab, Los Alamos, N.M. 1993.

[13] C.M. Brislawn, J.N.Bradley,R.J.Onyshczak, and T.Hopper. The FBI compressiom standard for digitized fingerprint images. In Proc. Of SPIE, Vol.2847, Denver, pages 344-355,aug.1996.

[14] K. Shashi,, K B Raja, R K Chhotaray, P. Sabyasachi, " DWT Based Fingerprint Recognition using Non Minutiae Features", IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 2, March 2011.

[15] A. Pokhriyal and S. Lehri, "A New Method of Fingerprint Authentication using 2-D wavelets", Journal of Theoretical and Applied Information Technology, vol. 13, no.2, pp. 131 - 138, March , 2010.

[16] | S. Grgic, M. Grgic, Member, IEEE, and Branka Zovko-Cihlar, Member, IEEE. Performance Analysis of Image Compression Using Wavelets. IEEE Transactions On Industrial Electronics, Vol. 48, No. 3, June 2001.

[17] ISO/IEC. ISOIIEC 15444-1. Inlixmation technolop - JPEG2000. image coding system. 2000.

[18] ISO/IEC JTC/SC29/WGII. N2507a. Generic Coding of Audio-Visual objects: Visual 14496-7. Final Draft IS. Atlantic City. Dec. 1998.

[19] ISO/IEC. International Standard DIS 10918. Digital Compression and Coding of Continuous-Tone Still Images.

[20] L. Kaur, S. Gupta, R.C. Chauhan, "Image Depoising using Wavelet Thresholding".

[21] D. L. Donoho, I. M. Johnstone, G. Kerkyacharian and D. Picard," Density Estimation by Wavelet Thresholding", The Annals of Statistics, Vol. 24, No. 2 (Apr., 1996), pp. 508-539

[22] Yansun Xu, John B. Weaver, Dennis M. Healy, Jr., and Jian Lu .Wavelet Transform Domain Filters: A Spatially Selective Noise Filtration Technique. IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 3, NO. 6, NOVEMBER 1994.

[23] A.J. Goldstein, L.D. Harmon, and A.B. Lesk, "Identification of Human Faces," proc. IEEE,May,1971,Vol.59, No.5,748-760.

[24] N. Nainl, A. Kumar, A. K. Mohapatra, R. Das, A. Kumar & N. C. Singh, "Face Recognition Using LDA With Wavelet Transform Approach", International Journal of Information Technology and Knowledge Management July-December 2011, Volume 4, No. 2, pp. 603-607.