Common Fixed Point Theorems in Soft S-Metric Spaces

Sneha A. Khandait[1], Ramakant Bhardwaj[2], Chitra Singh[3]
[1,3]Dept of Science, Rabindranath Tagore University, Bhopal (M.P) India.
[2]Dept. of Mathematics, Amity university, Kolkata (W.B.) India.

ABSTRACT
The aim to this paper is to define soft S-metric space and investigate some important properties. Also We established some fixed point theorem for two maps on soft S-metric spaces. Our results isgeneralized form of some previous well known results from S-metric spaces.

Keywords: Soft S-metric spaces, fixed points. AMS subject classification**:** 47H10, 54H25, 55M20.

I INTRODUCTION

In daily life, the problem in many fields deals with uncertain data and are not successfully modelled in classical mathematics. In 1999, the theory of soft sets initiated by Molodstov [15] as mathematical tool to handle uncertainty associated with real world data based problems and established the fundamental results of this new concept. The soft set is a parametrized family of subset of universal set. Majiet al.[16] introduced several operations in soft sets. They made theoretical study of the soft set theory and presented the application of soft set in decision making problem. Das and Samanta[7, 9, 10] introducing the concept of soft metric spaces which is based on soft point of soft sets. Fixed point problem for contractive mapping in metric spaces with partial order have been studied by many authors ([1],[5]). Jungck [11] gave the concept of fixed point for commuting map. In 2006, Z. Mustafa and B.I. Sims [17] introduced the notion of G-metric spaces which is generalization of metric spaces.

In 2012, S. Sedghi, N. Shobe and Aiouche [21] introduced the notion of S-metric which is generalization of G-metric spaces [11] and D^* metric [20]. Also, Sedghi and N.V. Dung [22] investigated some generalized fixed point theorems in S-metric spaces which is generalization of [21]. Jong Kyu Kim et al. [14] gave some fixed point theorems for two maps on complete S-metric spaces. Recently Mujeeb, Mohammad and Muhib[27] introduced the notion of common fixed point theorems for compatible and weakly compatible mapping in context of S-metric spaces. In the present paper we introduced the notion of soft S-metric spaces which is generalization of S-metric spaces and also, fixed point theorems for two mappings on complete soft S-metric spaces will be proved. Our result extends and generalized form of result of [14].

We recall some definitions will be needed in the sequel.

II PRELIMINARIES AND DEFINITIONS

(a) Definition 2.1. [15] Let A be a set of parameters and X be an initial universe. Let P(X) denote the power set of X. A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.

(b) Definition 2.2. [16] Let (F,A) and (G,A) be two soft sets over a common universe X.

(i) (F,A) is said to be null soft set, denoted by $\emptyset$, if for all $\lambda \in A$, $F(\lambda) = \emptyset$. (F,A) is said to an absolute soft set denoted by $\tilde{E}$, if for all $\lambda \in A$, $F(\lambda) = E$.

(ii) (F,A) is said to be a soft subset of (G,A) if for all $\lambda \in A$, $F(\lambda) \subseteq G(\lambda)$ and it is denoted by $(F,A) \tilde{\subseteq} (G,A)$. (F,A) is said to be a soft upper set of (G,A) if (G,A) is a soft subset of (F,A). We denote it by $(F,A) \tilde{\supseteq} (G,A)$. (F,A) and (G,A) is said to be equal if (F,A) is a soft subset of (G,A) and (G,A) is a soft subset of (F,A).

(iii) The intersection (H,A) of (F,A) and (G,A) over E is defined as $H(\lambda) = F(\lambda) \cap G(\lambda)$ for all $\lambda \in A$. We write $(F,A) \tilde{\cap} (G,A) = (H,A)$.

(iv) The union (H,A) of (F,A) and (G,A) over E is defined as $H(\lambda) = F(\lambda) \cup G(\lambda)$ for all $\lambda \in A$. We write $(F,A) \tilde{\cup} (G,A) = (H,A)$.

(v) The Cartesian product (H,A) of (F,A) and (G,A) over E denoted by $(H,A) = (F,A) \tilde{\times} (G,A)$, is defined as $H(\lambda) = F(\lambda) \times G(\lambda)$ for all $\lambda \in A$.

(vi) The difference (H,A) of (F,A) and (G,A) over E denoted by $(F,A) \tilde{/} (G,A) = (H,A)$, is defined as $H(\lambda) = F(\lambda) \backslash G(\lambda)$ for all $\lambda \in A$.

(vii) The complement of (F,A) is defined as $(F,A)^c = (F^c,A)$, where $F^c : A \to P(E)$ is a mapping given by $F^c(\lambda) = B \backslash F(\lambda)$ for all $\lambda \in A$. Clearly, we have $\widetilde{E^c} = \Phi$ and $\Phi^c = \tilde{E}$.

(c) Definition 2.3: [7,9] Let A be a set of parameters and $\mathbb{R}$ be the set of real numbers and $B(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$. Then a mapping $F : A \to B(\mathbb{R})$ is called a soft real set, denoted by (F,A). If specifically (F,A) is a singleton soft set, then after identifying (F,A) with the corresponding soft element, it will be called a soft real number. The set of all soft real numbers is denoted by $\mathbb{R}(A)$ and the set of non-negative soft real numbers by $\mathbb{R}(A)^*$.

Let $\tilde{r}$ and $\tilde{s}$ be two soft real numbers. Then the following statements hold:

(i) $\tilde{r} \tilde{\leq} \tilde{s}$, if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$, $\forall \lambda \in A$,

(ii) $\tilde{r} \tilde{<} \tilde{s}$, if $\tilde{r}(\lambda) < \tilde{s}(\lambda)$, $\forall \lambda \in A$,

(iii) $\tilde{r} \tilde{\geq} \tilde{s}$, if $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$, $\forall \lambda \in A$,

(iv) $\tilde{r} \tilde{>} \tilde{s}$, if $\tilde{r}(\lambda) > \tilde{s}(\lambda)$, $\forall \lambda \in A$,

(d) Definition 2.4: [9] Let X be a non-empty set and A be non-empty a parameter set. A mapping d: $\mathbb{SE}(\tilde{X}) \times \mathbb{SE}(\tilde{X}) \rightarrow \mathbb{R}(A)^*$ is said to be a soft metric on the soft set $\tilde{X}$ if d satisfies the following conditions:

(i) $d(\tilde{x}, \tilde{y}) \tilde{\geq} \tilde{0}$, for all $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$.

(ii) $d(\tilde{x}, \tilde{y}) = \bar{0}$ if and only if $\tilde{x} = \tilde{y}$.

(iii) $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x})$ for all $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$.

(iv) $d(\tilde{x}, \tilde{y}) \tilde{\leq} d(\tilde{x}, \tilde{z}) + d(\tilde{z}, \tilde{y})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} \tilde{X}$.

The soft $\tilde{X}$ with a soft metric d on $\tilde{X}$ is said to be a soft metric space and denoted by $(\tilde{X}, d, A)$ or $(\tilde{X}, d)$.

(e) Definition 2.5: [21] Let X be a nonempty set. An S-metric on X is a function $S: X \times X \times X \longrightarrow [0, \infty)$ that satisfies the following conditions holds for all $x, y, z, a \in X$.

(i) $S(x, y, z) \geq 0$,

(ii) $S(x, y, z) = 0$ iff $x = y = z$,

(iii) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

The pair (X, S) is called a S-metric space.

Example 2.1: [21] Let X=R the distance function $S: X^3 \longrightarrow [0, \infty)$ defined by

$S(x, y, z) = |x - z| + |y - z|$ for all $x, y, z, \in X$ is a S-metric on X.

Definition 2.2:[21] Let (X, S) be a S-metric space for $r > 0$ and $x \in X$, we define the open ball $B_S(x, r)$ and the closed ball $B_S[x, r]$ with centre x and radious r as follows:

$B_S(x, r) = \{y \in X: S(y, y, x) < r\}$,

$B_S[x, r] = \{y \in X: S(y, y, x) \leq r\}$.

The topology induced by the S-metric is the topology generated by the base of all open ball in X.

Definition 2.3:[14]Let(X, S) and (X', S') be two S-metric space. A function $f: (X, S) \longrightarrow (X', S')$ is said to be continuous at point $a \in X$ if for every sequence $\{x_n\}$ in X with $S(x_n, x_n, a) \longrightarrow 0$, $S'(f(x_n), f(x_n), f(a)) \longrightarrow 0$. We say that f is continuous on X if f is continuous at every point $a \in X$.

III SOFT S-METRIC SPACES

In this section, we introduce soft S-metric spaces and also we give some important results.

Let $\tilde{X}$ be the absolute soft set, A be a non-empty set of parameters and $\mathbb{SE}(\tilde{X})$ be the collection of all soft points of $\tilde{X}$. Let $\mathbb{R}(A)^*$ denote the set of all non-negative soft real numbers.

(a) Definition 3.1: Let X be a non-empty set and $\tilde{X}$ be absolute soft set. A mapping

$S: \mathbb{SE}(\tilde{X}) \times \mathbb{SE}(\tilde{X}) \times \mathbb{SE}(\tilde{X}) \longrightarrow [0, \infty)$ is said to be a soft S-metric on $\tilde{X}$ if S satisfies the following axioms holds for all $\tilde{x}, \tilde{y}, \tilde{z}, \tilde{a} \tilde{\in} \tilde{X}$,

(S_1) $\Theta \tilde{\leq} S(\tilde{x}, \tilde{y}, \tilde{z})$,

(S_2) $S(\tilde{x}, \tilde{y}, \tilde{z}) = \Theta$ iff $\tilde{x} = \tilde{y} = \tilde{z}$

(S_3) $S(\tilde{x}, \tilde{y}, \tilde{z}) \tilde{\leq} S(\tilde{x}, \tilde{x}, \tilde{a}) + S(\tilde{y}, \tilde{y}, \tilde{a}) + S(\tilde{z}, \tilde{z}, \tilde{a})$

Then, the soft set $\tilde{X}$ with a soft S-metric$S$ on $\tilde{X}$ is called a soft S-metric space and is denoted by $(\tilde{X}, S, A)$ or $(\tilde{X}, S)$.

Then S is called soft S-metric on $\tilde{X}$ and $(\tilde{X}, S)$ is called soft S-metric space.

Example 3.1: Let A be a finite set of parameters, Let $\tilde{X} = \mathbb{R}^3(A)$,

$(S, A) = SS\{(\tilde{x}, \tilde{y}, \tilde{z}) \tilde{\subset} \tilde{X}: \tilde{x}, \tilde{y}, \tilde{z} \tilde{\geq} \tilde{0}\}$, the function $S: \tilde{X}^3 \longrightarrow [0, \infty)$ such that,

$S(\tilde{x}, \tilde{y}, \tilde{z}) = |\tilde{x} - \tilde{z}| + |\tilde{y} - \tilde{z}|$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in \tilde{X}$.

The $(\tilde{X}, S, A)$ is a soft S-metric spaces.

(b) Definition 3.2: Let $(\tilde{X}, S)$ be soft S-metric space. Let $\{\widetilde{x_n}\}$ be a sequence of soft elements in $\tilde{X}$ and $\tilde{x} \in \tilde{X}$. $\{\widetilde{x_n}\}$ converges to $\tilde{x}$ if $S(\widetilde{x_n}, \widetilde{x_n}, \tilde{x}) \longrightarrow 0$ as $n \longrightarrow \infty$. That is, for each$\tilde{\varepsilon} \tilde{>} 0$, there exists $n_0 \in \mathbb{N}$, such that for all $n \tilde{\geq} n_0$, we have, $S(\widetilde{x_n}, \widetilde{x_n}, \tilde{x}) \tilde{<} \tilde{\varepsilon}$.We denote this$\lim_{n\to\infty} \widetilde{x_n} \longrightarrow \tilde{x}$.

(c) Definition 3.3: Let $(\tilde{X}, S)$ be soft S-metric space. Let $\{\widetilde{x_n}\}$ be a sequence of soft elements in $\tilde{X}$. A sequence $\{x_n\}$ Cauchy sequence if $S(\widetilde{x_n}, \widetilde{x_n}, \widetilde{x_m}) \longrightarrow 0$ as $n, m \longrightarrow \infty$. That is, for each $\tilde{\varepsilon} \tilde{>} 0$, there exists $n_0 \in \mathbb{N}$ (there is natural number $\mathbb{N}$), such that for all $n, m \tilde{\geq} n_0$, we have, $S(\widetilde{x_n}, \widetilde{x_n}, \widetilde{x_m}) \tilde{<} \tilde{\varepsilon}$.

(d) Definition 3.4: Let $(\tilde{X}, S)$ be soft S-metric space. If every Cauchy sequence of soft elements in $\tilde{X}$ is convergent in $\tilde{X}$, then $(\tilde{X}, S)$ is called complete soft S-metric space.

(e) Definition 3.5: Letτ be the soft set of all A$\tilde{\subset}$ $\tilde{X}$ with $\tilde{x} \in A$ and there exists $r \tilde{>} 0$ such that $B_S(\tilde{x}, r) \tilde{\subset} A$. Then τ is a topolpgy on $\tilde{X}$.

(f) Definition 3.6: Let $(\tilde{X}, S)$ be soft S-metric space. A mapping T:$\tilde{X} \longrightarrow \tilde{X}$ is called contraction if $S(T\tilde{x}, T\tilde{x}, T\tilde{y}) \tilde{\leq} \alpha. S(\tilde{x}, \tilde{x}. \tilde{y})$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$ with $\alpha \in [0,1)$.

Some basic results which is generalized form of S-metric space[21]:

- **(i) Basic result 3.1:**Let $(\tilde{X}, S)$ be soft S-metric space. Limit of the convergent sequence $\{x_n\}$ in soft S-metric space $(\tilde{X}, S)$ is unique.
- **(ii) Basic result 3.2:** Soft S-metric$(\tilde{X}, S)$ is jointly continuous on all three variables.
- **(iii) Basic result 3.3:** In soft S-metric space we have, $S(\tilde{x}, \tilde{x}. \tilde{y}) = S(\tilde{y}, \tilde{y}. \tilde{x})$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$.
- **(iv) Basic result 3.4:**Let $(\tilde{X}, S)$ be soft S-metric space. If there exists sequence $\{\widetilde{x_n}\}$ and $\{\widetilde{y_n}\}$ be the soft elements in $\tilde{X}$, such that $\lim_{n\to\infty} \widetilde{x_n} \longrightarrow \tilde{x}$ and $\lim_{n\to\infty} \widetilde{y_n} \longrightarrow \tilde{y}$, then

$\lim_{n\to\infty} S(\widetilde{x_n}, \widetilde{x_n}, \widetilde{y_n}) = S(\tilde{x}, \tilde{x}. \tilde{y})$.

- **(v) Basic result 3.5:**Let $(\tilde{X}, S)$ be soft S-metric space and suppose that sequence $\{\widetilde{x_n}\}$ and $\{\widetilde{y_n}\}$ are S-convergent to $\tilde{x}, \tilde{y}$ respectively. Then we have

$\lim_{n\to\infty} sup\, S(\widetilde{x_n}, \tilde{z}, \widetilde{y_n}) \leq S(\tilde{z}, \tilde{z}. \tilde{x}) + S(\tilde{x}, \tilde{x}. \tilde{y})$.

In particular, if $\tilde{x} = \tilde{y}$, then we have $\lim_{n\to\infty} sup\, S(\widetilde{x_n}, \tilde{z}, \widetilde{y_n}) \leq S(\tilde{z}, \tilde{z}. \tilde{x})$.

Proof: Let $\lim_{n\to\infty} \widetilde{x_n} \longrightarrow \tilde{x}$ and $\lim_{n\to\infty} \widetilde{y_n} \longrightarrow \tilde{y}$. Then for each $\tilde{\varepsilon} > 0$ there exists $n_1, n_2 \tilde{\in} \mathbb{N}$ such that for all $n \geq n_1$,

$$S(\widetilde{x_n}, \widetilde{x_n}, \tilde{x}) < \frac{\tilde{\varepsilon}}{2}$$

for all $n \geq n_2$,

$S(\widetilde{y_n}, \widetilde{y_n}, \tilde{y}) < \frac{\tilde{\varepsilon}}{4}$.

If set $n_0 = \max\{n_1, n_2\}$, then for every $n \geq n_0$ by condition (3) of soft S-metric space, we have

$S(\widetilde{x_n}, \tilde{z}, \widetilde{y_n}) \leq S(\widetilde{x_n}, \widetilde{x_n}, \tilde{x}) + S(\tilde{z}, \tilde{z}. \tilde{x}) + S(\widetilde{y_n}, \widetilde{y_n}, \tilde{x})$

$\leq S(\widetilde{x_n}, \widetilde{x_n}, \tilde{x}) + S(\tilde{z}, \tilde{z}. \tilde{x}) + 2S(\widetilde{y_n}, \widetilde{y_n}, \tilde{y}) + S(\tilde{x}, \tilde{x}. \tilde{y})$

On taking the upper limit as $n \longrightarrow \infty$ in the above inequality, we get the first desired result.

The second result is obvious.

This result is generalized form of S-metric space [13].

Invariant points on soft S-metric space:

- **(vi) Definition 3.7:** Let $(\tilde{X}, S)$ be soft S-metric space and T:$(\tilde{X}, S) \longrightarrow (\tilde{X}, S)$ be mapping. If there exists soft elements $\widetilde{x_0} \in \tilde{X}$ such that $T\widetilde{x_0} = \widetilde{x_0}$, then the $\widetilde{x_0}$ is called fixed element in T.
- **(vii) Definition 3.8:** Let $(\tilde{X}, S)$ be soft S-metric space and T:$(\tilde{X}, S) \longrightarrow (\tilde{X}, S)$ be mapping. For every $\widetilde{x_0} \in \tilde{X}$, we can construct the sequence $\{\widetilde{x_n}\}$ of soft elements by choosing $\widetilde{x_0}$ and continuing by;

$\widetilde{x_1} = T\widetilde{x_0},\ \widetilde{x_2} = T\widetilde{x_1} = T^2\widetilde{x_0}, \ldots\ldots\ldots\ldots\ldots\ \widetilde{x_n} = T^n\widetilde{x_0}, \ldots..$

We say that the sequence $\{\widetilde{x_n}\}$ is constructed by iteration method.

IV MAIN RESULTS

Let ψ denote the class of all functions ψ: $R^+ \longrightarrow R^+$ such that ψ is non-decreasing, continuous and $\sum_{n=1}^{\infty} \psi^n(t) < \infty\ for\ all\ t > 0$. It is clear that $\psi^n(t) \longrightarrow 0$ as $n \longrightarrow \infty for\ all\ t > 0$ and therefore we have $\psi(t) < t \forall\ t > 0$.

Theorem 4.1:$\tilde{X}$ be a soft set. Let the metric space $(\tilde{X}, S)$ be complete soft S-metric space and consider the map P, Q: $\tilde{X} \longrightarrow \tilde{X}$ be mappings satisfies the following condition :

1) $P(\tilde{X}) \tilde{\subseteq} Q(\tilde{X})\ and\ either\ P(\tilde{X})\ or\ Q(\tilde{X})\ is\ closed\ subset\ of\ \tilde{X}$,

2) The pair (P, Q) is weakly compatible ;

3) $S(P\tilde{x}, P\tilde{y}, P\tilde{z}) \leq \psi(\max\{S(Q\tilde{x}, Q\tilde{y}, Q\tilde{z}), h_1 S(Q\tilde{z}, P\tilde{x}, P\tilde{z}), h_2 S(Q\tilde{z}, P\tilde{y}, P\tilde{z}), h_3 S(Q\tilde{z}, P\tilde{y}, P\tilde{z})\})$

[4.1.1]

For all $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} \tilde{X}$ and $0 < h_1, h_2, h_3 < 1$ where $\psi \tilde{\in} \phi$.

Then the map $P\ and\ Q$have unique common fixed point. If Q is continuous at the fixed point $\tilde{r}$, then P is also continuous at $\tilde{r}$.

Proof: Let $\widetilde{x_0} \tilde{\in} \tilde{X}$. We define a sequence $\{\widetilde{y_n}\}\ in\ \tilde{X}$ such that

$\widetilde{y_n} = P\widetilde{x_n} = Q\widetilde{x_{n+1}}$ for $n = 0,1,2, \ldots..$

And let $\widetilde{d_{n+1}} = S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}})$. Then we have

$\widetilde{d_{n+1}} = S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}})$

$$= S(P\widetilde{x_n}, P\widetilde{x_n}, P\widetilde{x_{n+1}})$$

$$S(P\widetilde{x_n}, P\widetilde{x_n}, P\widetilde{x_{n+1}}) \leq \psi(\max\{S(Q\widetilde{x_n}, Q\widetilde{x_n}, Q\widetilde{x_{n+1}}), h_1 S(Q\widetilde{x_{n+1}}, P\widetilde{x_n}, P\widetilde{x_{n+1}}),$$

$h_2 S(Q\widetilde{x_{n+1}}, P\widetilde{x_n}, P\widetilde{x_{n+1}})$, $h_3 S(Q\widetilde{x_{n+1}}, P\widetilde{x_n}, P\widetilde{x_n})\})$
$= \psi(\max\{S(\widetilde{y_{n-1}}, \widetilde{y_{n-1}}, \widetilde{y_n}), h_1 S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}}), h_2 S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}})$
$h_3 S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_n})\})$
$\leq \psi(\max\{\widetilde{d_n}, h_1\widetilde{d_{n+1}}, h_2\widetilde{d_{n+1}}\})$

Thus $\widetilde{d_{n+1}} \leq \psi(\widetilde{d_n})$, $n = 1,2, \ldots\ldots$ Hence we have

$S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}}) \leq \psi(S(\widetilde{y_{n-1}}, \widetilde{y_{n-1}}, \widetilde{y_n}))$
$\leq \psi^2 . S(\widetilde{y_{n-2}}, \widetilde{y_{n-2}}, \widetilde{y_{n-1}})$

Continuing this process, we get

$S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}}) \leq \psi^n S(\widetilde{y_0}, \widetilde{y_0}, \widetilde{y_1})$ (4.1.2)

Hence for every $m > n$ by using (S_3), we have,

$S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_m}) \leq S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}}) + S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}}) + S(\widetilde{y_{n+1}}, \widetilde{y_{n+1}}, \widetilde{y_{n+2}})$
$\leq 2\, S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}}) + S(\widetilde{y_{n+1}}, \widetilde{y_{n+1}}, \widetilde{y_{n+2}})$
$\leq 2\, [S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_{n+1}}) + S(\widetilde{y_{n+1}}, \widetilde{y_{n+1}}, \widetilde{y_{n+2}})] + S(\widetilde{y_{n+2}}, \widetilde{y_{n+2}}, \widetilde{y_{n+3}})$
$\vdots$
$\leq 2\sum_{i=n}^{m-2} S(\widetilde{y_\iota}, \widetilde{y_\iota}, \widetilde{y_{\iota+1}}) + (\widetilde{y_{m-1}}, \widetilde{y_{m-1}}, \widetilde{y_m})$
$\leq 2\, [\psi^n(S(\widetilde{y_0}, \widetilde{y_0}, \widetilde{y_1})) + \psi^{n+1}(S(\widetilde{y_0}, \widetilde{y_0}, \widetilde{y_1})) + \ldots + \psi^{m-1}(S(\widetilde{y_0}, \widetilde{y_0}, \widetilde{y_1}))$
$= 2\sum_{i=n}^{m-1} \psi^i(S(\widetilde{y_0}, \widetilde{y_0}, \widetilde{y_1}))$.

Since $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$, $S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_m}) \longrightarrow 0$ as $n \longrightarrow \infty$. So, for each $\tilde{\varepsilon} > 0$, there exists $n_0 \,\tilde{\in}\, \mathbb{N}$ such that for each $n, m \geq n_0$
$S(\widetilde{y_n}, \widetilde{y_n}, \widetilde{y_m}) < \tilde{\varepsilon}$.

This shows that $\{\widetilde{y_n}\}$ is Cauchy sequence in $\tilde{X}$. Since $\tilde{X}$ is complete, there exists $\tilde{r} \,\tilde{\in}\, \tilde{X}$ such that $\lim_{n\to\infty} \widetilde{y_n} = \tilde{r}$ and
$\tilde{r} = \lim_{n\to\infty} \widetilde{y_n} = \lim_{n\to\infty} P\widetilde{x_n} = \lim_{n\to\infty} Q\widetilde{x_{n+1}}$.

Let $Q(\tilde{X})$ be a closed subset of $\tilde{X}$. Then there exists $\tilde{v} \,\tilde{\in}\, \tilde{X}$ such that $Q\tilde{v} = \tilde{r}$.

We prove that $P\tilde{v} = \tilde{r}$. Since,

$S(P\tilde{v}, P\tilde{v}, P\widetilde{x_n}) \leq \psi(\max\{S(Q\tilde{v}, Q\tilde{v}, Q\widetilde{x_n}), h_1 S(Q\widetilde{x_n}, P\tilde{v}, P\widetilde{x_n}),$
$h_2 S(Q\widetilde{x_n}, P\tilde{v}, P\widetilde{x_n}), h_3 S(Q\widetilde{x_n}, P\tilde{v}, P\tilde{v})\})$
$= \psi(\max\{S(\tilde{r}, \tilde{r}, \widetilde{y_{n-1}}), h_1 S(\widetilde{y_{n-1}}, P\tilde{v}, \widetilde{y_n}), h_2 S(\widetilde{y_{n-1}}, P\tilde{v}, \widetilde{y_n})$
$h_3 S(\widetilde{y_{n-1}}, P\tilde{v}, P\tilde{v})\})$

On taking upper limit as $n \longrightarrow \infty$ in the above inequality, by basic result 3.5, we get

$S(P\tilde{v}, P\tilde{v}, P\widetilde{x_n})$
$\leq \psi(\max\{0, h_1 \lim_{\to\infty} \quad (\widetilde{\quad}_{-1}, \ \tilde{}\ , \tilde{}\ ),$
$h_2 \lim_{\to\infty} \quad (\widetilde{\quad}_{-1}, \ \tilde{}\ , \tilde{}\ ), h_3(0)\})$
$\leq \quad (\max\{0, h_1 \ (\ \tilde{}\ , \ \tilde{}\ , \tilde{}\ ), h_2 \ (\ \tilde{}\ , \ \tilde{}\ , \tilde{}\ ), 0\})$
$\leq \max\{h_1, h_2\} \ (\ \tilde{}\ , \ \tilde{}\ , \tilde{}\ )$.

This implies that $1 \leq \max\{h_1, h_2\}$, which is contradiction. Hence, from ( ) < ∀ > 0,
We have ~ = ~ = ~.

By the weak compatibility of the pair (,), we have ~ = ~ and hence ~ = ~. Now, we have to prove that ~ = ~. Consider ~ ≠ ~. Then

(~, ~, ~) ≤ (max{ (~, ~, ~), h_1 (~, ~, ~),
h_2 (~, ~, ~), h_3 (~, ~, ~)})
= (max{ (~, ~, $_{-1}$), h_1 ($_{-1}$, ~, ~), h_2 ($_{-1}$, ~, ~)
h_3 ($_{-1}$, ~, ~)})

On taking upper limit as ⟶ ∞ in the above inequality, we get

(~, ~, ~)
≤ (max{ (~, ~, ~), h_1 lim$_{\to\infty}$ ($_{-1}$, ~, ~),
h_2 lim$_{\to\infty}$ ($_{-1}$, ~, ~), h_3 ($_{-1}$, ~, ~)})
≤ (max{0, h_1 (~, ~, ~), h_2 (~, ~, ~), 0})
≤ max{h_1, h_2} (~, ~, ~).

Since, ( ) < ∀ > 0, we have ~ = ~ = ~. Hence ~ is common fixed point of .

Let ~′ is another common fixed point of . Then,
(~, ~, ~′) = (~, ~, ~′)
≤ (max{ (~, ~, ~′), h_1 (~′, ~, ~′), h_2 (~′, ~, ~′), h_3 (~′, ~, ~′)})

If (~, ~, ~′) ≤ ((~, ~, ~′)), then (~, ~, ~′) ≤ ((~, ~, ~′)) < (~, ~, ~′) which is contradiction.
Therefore, we have ~ = ~′. If (~, ~, ~′) < h (~′, ~, ~′), then

(~, ~, ~′) < h (~′, ~, ~′)
≤ (2 (~′, ~′, ~′) + (~, ~, ~′)
= (~, ~, ~′),

Where = max{ $_1$, $_2$}, this is also contradiction. Therefore, we have ~ = ~′. Hence ~ is unique common fixed point of .

Now, we prove continuity of the mapping in soft S-metric space.

Consider sequence { ~ } in ~ such that { ~ } is convergent to ~. Then we have

(~, ~, ~) ≤ (max{ (~, ~, ~), $_1$ (~, ~, ~), $_2$ (~, ~, ~), $_3$ (~, ~, ~)})

On taking upper limit as → ∞ in the above inequality, from the continuity of at a point ~, we get

$\lim_{\to\infty}$ (~, ~, ~) = $\lim_{\to\infty}$ (~, ~, ~)

≤ (max{ $\lim_{\to\infty}$ (~, ~, ~), $_1 \lim_{\to\infty}$ (~, ~, ~), $_2 \lim_{\to\infty}$ (~, ~, ~), $_3$ (~, ~, ~)})

≤ (max{0, $_1 \lim_{\to\infty}$ (~, ~, ~), $_2 \lim_{\to\infty}$ (~, ~, ~), 0})

≤ max{ $_1$, $_2$} (~, ~, ~).

Since,

$_1 \lim_{\to\infty}$ (~, ~, ~) ≤ $_1 \lim_{\to\infty}$ (~, ~, ~,) + $_1 \lim_{\to\infty}$ (~, ~, ~) + $_1 \lim_{\to\infty}$ (~, ~, ~)

And

$_2 \lim_{\to\infty}$ (~, ~, ~) ≤ $_2 \lim_{\to\infty}$ (~, ~, ~,) + $_2 \lim_{\to\infty}$ (~, ~, ~) + $_2 \lim_{\to\infty}$ (~, ~, ~)

We have,

$\lim_{\to\infty}$ (~, ~, ~) ≤ max{ $_1$, $_2$} $\lim_{\to\infty}$ (~, ~, ~).

Which is implies that $\lim_{\to\infty}$ (~, ~, ~) = 0. Then, we conclude that is continuous at ~.

Theorem 4.2: ~ be a soft set. Let the complete soft S-metric space (~,) and consider the map , : ~ ⟶ ~ be continuous and be commutative with . If for every ∈̃ , the following conditionsatisfing:

1) (~) ⊆̃ (~) (~) (~) ~,

2) The pair (,) is weakly compatible ;

3) (~, ~, ~) ≤ (max{ (~, ~, ~), $_1$ (~, ~, ~), $_2$ (~, ~, ~), $_3$ (~, ~, ~)})

[4.1.1]

For all ~, ~, ~ ∈̃ ~ and $0 < {}_1, {}_2, {}_3 < 1$ where ∈̃ .

Then the map Q have unique common fixed point ~ ∈ ~. Further, if is continuous at the fixed point ~, then is also continuous at ~.

Proof: The proof is similar to the proof of theorem 4.1.

V CONCLUSION

In this paper the concept of soft S-metric space via soft elements. Some fixed point theorems for two mapping on complete soft S-metric space was also discussed. There are ample scopes for further research on soft S-metric space.

REFERENCES

[1] R.P. Agarwal, M. A. El-Gebeily, D. O'regan, Generalized contractions in partially ordered metric spaces, Appl. Anal., 87(2008), 109-116.

[2] M.AamriandD.EI.Moutawakil,Somenewcommon fixedpointtheoremsunderstrictcontractiveconditions, Journal of Mathematical Analysis and Applications,270(2002), 181-188.

[3] M. Abbas, T. Nazir and P. Vetro, Common fixed point results for three mappings in G-metric space, Faculity of Science and Mathematics, University of Nis Serbia,25(2011), 1-17.

[4] B.S. Choudhury, S. Kumar, Asha and K. Das, Some fixed point theorems in G-metric spaces, Mathematical Sciences Letters,1(2012), 25-31.

[5] Lj. ´Ciri´c, D. Mihet, R. Saadati, Monotone generalized contractions in partiality ordered probabilistic metric spaces, Topology Appl., 17(2009), 2838-2844.

[6] R.Chugh, T.Kadian, A.Rani, B.E.Rhoades, Property P in G-metric spaces, Fixed Point Theory Appl. Vol.2010, Article ID 401684.

[7] S. Das, S. K. Samanta, Soft real sets, soft real numbers and their properties, J. Fuzzy Math. 20 (3) (2012) 551–576.

[8] S. Das, S. K. Samanta, On soft complex sets and soft complex numbers, J. Fuzzy Math. 21 (1) (2013) 195–216.

[9] S. Das, S. K. Samanta, On soft metric spaces, J. Fuzzy Math. 21 (3) (2013) 707–734.

[10] S. Das, S. K. Samanta, Soft metric, Ann. Fuzzy Math. Inform. 6 (2013), no. 1, 77–94.

[11] G. Jungck, Commuting mappings and xed points, Amer. Math. Monthly 83 (1976) 261263.

[12] G. Jungck, Compatible mappings and common fixed points, Int. J. Math. Math. Sci.,9(1986), 771-779.

[13] G. Jungck, Common fixed points for non-continuous non-self mappings on non-metric spaces, Far East J. Math. Sci.,4(1996), 199-212.

[14] Jong K. Kim, S. Sedghi, A. Gholidahneb and M. Rezaee, Fixed point theorems in S-metric spaces, East Asian math. J. Vol.32(2016), No.5, PP.677-684.

[15] D. Molodtsov, Soft set theory–first results, Comput. Math. Appl. 37 (4) (1999) 19–31. doi:10.1016/S0898-1221(99)00056-5.

[16] P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (4) (2003) 555–562. doi:10.1016/S0898-1221(03)00016-6.

[17] Z.MustafaandB.Sims,Anewapproachtogeneralizedmetricspaces,JournalofNonlinearConvexAnalysis, 7(2006), 289-297.

[18] Z. Mustafa, H. Obiedat and F. Awawdeh, Some fixed point theorem for mapping on complete G-metric spaces, Fixed Point Theory and Applications, 2008 (2008), Article ID 189870.

[19] R.P. Pant, Common fixed points of four maps, Bull. Calcutta Math. Soc. 90 (1998), 281-286.

[20] S. Sedghi, N. Shobe, H. Zhou, A common fixed point theorem in D∗-metric spaces, Fixed Point Theory Applications, 2007 (2007), Article ID 27906.

[21] S. Sedghi, N. Shobe, A. Aliouche, A generalization of fixed point theorem in S-metric spaces, Mat. Vesnik, 64(2012), 258266.

[22] S. Sedghi and V.N. Dung, Fixed point theorems on S-metric spaces, Mat. Veshik,66(2014), 113-124.

[23] S. Sedghi and I. Altun, N. Shobe, M.A. Shalahshour, Some properties of S-metric spaces and foxed point results, Kyungpook Math J., 54(2014), 113-122.

[24] S. Sedghi and N. Shobe, T. Dosenovic, Fixed point results in S-metric spaces, Nonlinear Functional Anal. And Appl., 20(2015), 55-67.

[25] S.Sedghi,N.Shobe,H.Zhou,AcommonfixedpointtheoreminD∗metricspaces,FixedPointTheoryAppl. 2007 (2007), Article ID 27906.

[26] W. Shatanawi, Fixed point theory for contractive mappings satisfying **Φ-** maps in G-metric spaces, Fixed Point Theory Appl. (2010), Article ID 181650.

[27] M. Rahman, M. Sarwar and Muhib UR Rahman, Some common fixed point theorems on S-metric spaces, J. fixed point theory, 2017, 2017:2 ISSN:2052-5338.