

## Uncertain Load Models and Their Representation in Power Systems

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### ABSTRACT

*Various uncertainties exist in the power systems and one of them is load uncertainty. As the power system network is undergoing various market pressures these days, it becomes very important to tackle the uncertainties. Due to the penetration of the distributed generation at the load ends in the system, the load bus uncertainty becomes a matter of serious concern. In this paper, the handling and representation of various uncertain load models are given. The affect of uncertainties in the system has been observed by a dynamic load flow program and severe case has been identified by a sensitivity index namely absolute minimum eigen value of load flow Jacobian. The proposed scheme is implemented on IEEE-14 bus system.*

**Keywords:** Uncertain loads, L-Index, dynamic load flow

### I INTRODUCTION

Various uncertainties exist in a power system and one of them is the load uncertainty. These uncertainties need to be tackled carefully while performing any kind of analysis in the power system. Due to the increasing nonlinear and dynamic loads, Distributed Generation units, unplanned outages, and other contingency events, the penetration level of uncertainty at the load end increases. The primary objective of the power system operators and planners is to make the operation of the system adequately within acceptable limits of the operating quantities. The conventional approach for the analysis purpose of the power system were based on deterministic methods but these days, the uncertainties in the complex systems are tackled by the probabilistic methods which bank upon the development of uncertain load models and dynamic load flow programs. In [1, 2], the estimation and representation for the complex analysis of the system are shown. The commonly used techniques for uncertainty and sensitivity analysis are presented in [2] using response surface analysis, Monte Carlo analysis, differential analysis, fast probability integration, Fourier amplitude sensitivity test, and Sobol’s variance decomposition. In this paper, the uncertain load models namely ‘Normal distribution, Exponential distribution, Gamma distribution, Beta distribution, Deviation from the base case loading and Lognormal distributed load models are presented. L indexes and the load flow results corresponding to

critical case are given in results and discussion section.

This paper is organized as follows: Section-I is introductory part, section-II deals with uncertain load models, the results and discussions are given in section III. Section IV contains the concluding remarks preceded by reference section.

### II UNCERTAIN LOAD MODELS

The proposed scheme is implemented on IEEE-14 bus system. Refer Appendix of this paper for details.

#### (a) Normal distributed load

Bus no. 4 is selected for normally distributed load modelling. This load modeling is done as follows [3,4]

$$f(P_D) = \frac{e^{-\frac{(P_D-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\Pi}}$$

(i)

$$-\infty \leq P_D \leq \infty$$

$$\sigma > 0$$

$$P_{Di}(\Psi_N) = \text{Normrnd}(\mu, \sigma, m, n) \text{ (ii)}$$

Where:

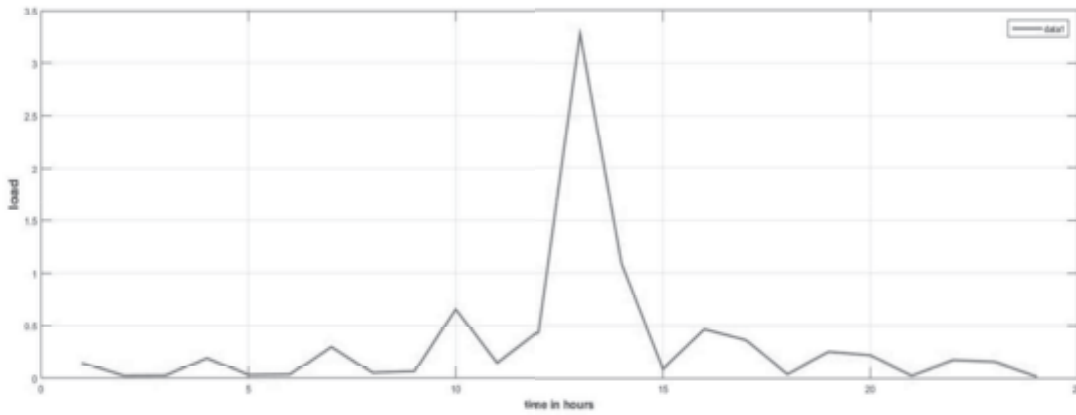
$P_D$  is the rated load at bus no. 4

$\mu=20$

$\sigma=2$

$m \times n$  is a  $24 \times 1$  array

The load distribution curve is shown in figure 1.



**Fig.1 normally distributed load at bus no.4**

**(b) Exponential distributed load**

Bus number 5 has been selected for exponential load modelling. This load modelling is done as follows:

$$f(P_D) = \frac{e^{-\frac{(P_D - \mu)}{b}}}{b} \tag{iii}$$

$$P_D \geq \mu$$

$$b > 0$$

Exponentially distributed real power load is at bus no. 5 with b=1.8 is shown in figure 2

**(c) Lognormal Distribution**

Bus number 12 has been selected for exponential load modelling. This load modelling is done as follows:

$$f(P_D) = \frac{e^{-\frac{\left(\ln\left(\frac{P_D - \mu}{m}\right)\right)^2}{2\sigma^2}}}{\sigma(P_D - \mu)\sqrt{2\Pi}} \tag{iv}$$

$$P_D \geq \mu$$

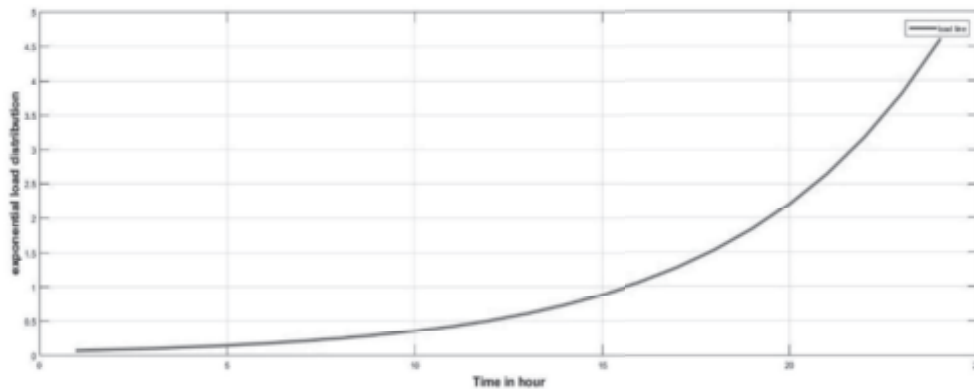
$$\sigma > 0$$

where

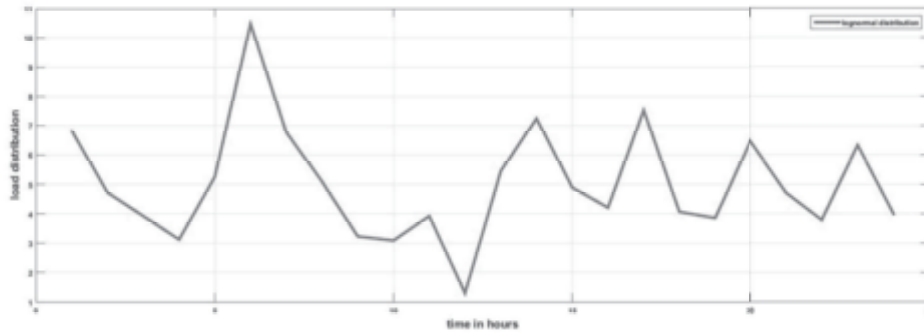
m: The scale parameter

ln: The natural logarithm

Lognormally distributed real power load is at bus i=12 with  $\mu=0.5$  and  $\sigma=0.3$  and its distribution is shown in figure 2



**Fig.2 exponentially distributed load at bus no.5**



**Fig.3 lognormally distributed load at bus no.12**

**(d) Gamma Distribution**

Bus number 11 has been selected for exponential load modelling. This load modelling is done as follows:

$$f(P_D) = \frac{(P_D - \mu)^{a-1}}{b^a \Gamma(a)} e^{-\left(\frac{P_D - \mu}{b}\right)} \quad (v)$$

$$P_D \geq \mu$$

$$a > 0$$

$$b > 0$$

Gamma distributed real power load is at bus i=12 with  $\mu=0.4$  and shown in figure 4.

$$d \leq P_D \leq c$$

$$a > 0$$

$$b > 0$$

where

a,b: The shape parameters

**(e) Deviation from the base loading**

Bus no. 10 is selected for this load modelling and it has been done as follows:

We took a load adjustment factor ( $\lambda$ ) which increases the load uniformly from 0 to 1.5 of rated load value and it is shown in figure 5.

$$P_{Di}(\Psi_{db}) = P_{Di}^0 (1 + \lambda) \quad (vi)$$

**(f) Beta Distribution**

Bus no. 7 is chosen for Beta distribution load modelling it has been done as follows

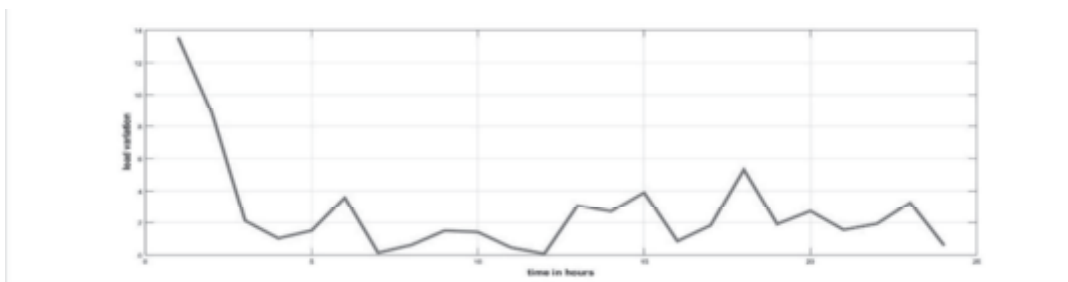
$$f(P_D) = \frac{(P_D - d)^{a-1} (c - P_D)^{b-1}}{B(a,b)(c-d)^{a+b-1}} = \frac{\Gamma(a+b)(P_D - d)^{a-1} (c - P_D)^{b-1}}{\Gamma(a)\Gamma(b)(c-d)^{a+b-1}} \quad (vii)$$

c: The upper bound

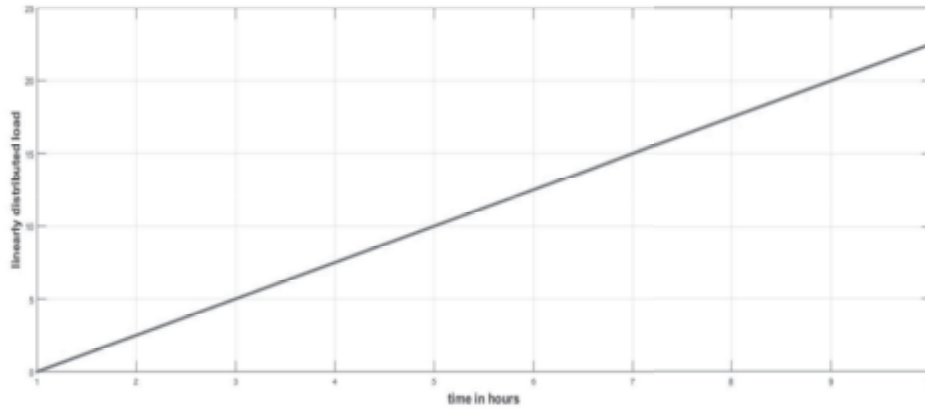
d: The lower bound

B (a, b): The beta function

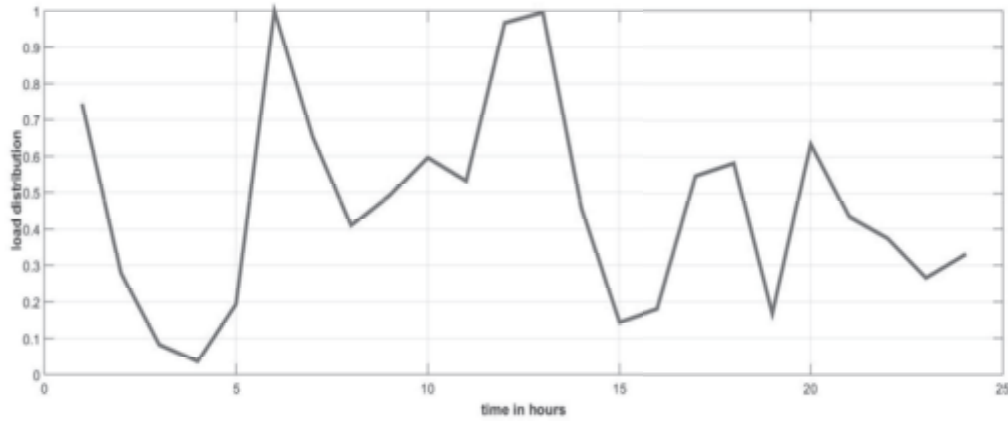
Here, the beta distributed load model is shown in figure 6.



**Fig.4 Gamma distributed load at bus no.11**



**Fig.5 Gamma distributed load at bus no.10**



**Fig.6 Beta distributed load at bus no.10**

**III RESULTS AND DISCUSSIONS**

Corresponding to above mentioned uncertain inputs a critical case is identified by a sensitivity index the results are discussed here. The L-index [5] for all the lines of IEEE-14 bus system for the critical case has been calculated by using the equation(viii) and shown in table 1 and the critical values which are close to unity are highlighted. Where:

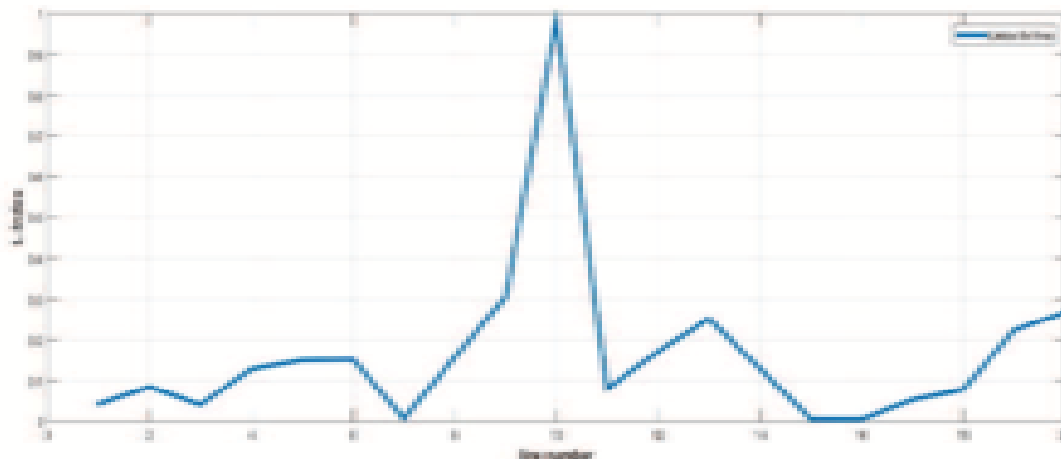
- $X_{ij}$ : Line reactance,
- $P_j$ : Active power at the receiving end
- $Q_j$ : Reactive power at the receiving end
- $V_i$ : Sending end voltage

$$L_{ij} = \frac{4[(P_j X_{ij} - Q_j R_{ij})^2 + (P_j R_{ij} + Q_j X_{ij})^2] V_i^2}{V_i^4} \tag{viii}$$

Any line in a system that exhibits  $L_{ij}$  closed to unity indicates that the line is approaching its stability limit and hence may lead to system violation.  $L_{ij}$  should always be less than unity in order to maintain a stable system.  $L_{ij}$  is termed as voltage stability index of the line. At collapse point, the value of  $L_{ij}$  will be unity. Based on voltage stability indices, voltage collapse can be accurately be predicted. The lines having high value of the index can be predicted as the critical lines, which contribute to voltage collapse. At or near the collapse point, voltage stability index of one or more-line approach to unity. This method is used to assess the voltage stability. Line voltage stability index is used to calculate the proximity of the operating point to voltage collapse.

**Table -1**  
**L index of all the lines**

<b>Line no.</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>L-index</b>	0.0427	0.0842	0.0409	0.1284	0.1503	0.1518	0.0067	0.1573	0.26712
<b>Line no.</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>
<b>L-index</b>	<b>0.3057</b>	<b>0.9850</b>	0.0797	0.1717	0.2521	0.1296	0.0061	0.0058	0.0530
<b>Line no.</b>	<b>19</b>	<b>20</b>	<b>Table shows the L-index for all the lines of IEEE-14 bus system</b>						
<b>L-index</b>	0.0778	0.2245							



**Fig.7 Variation of L-index**

**Table 2**  
**Load Flow Results Corresponding to Critical Case**  
**Min Eigenvalue = 0.5281**

Bus No.	Voltage Mag. (p.u.)	Active Load (pu)	Reactive Load (pu)	Active Generation (pu)	Reactive Generation (pu)
1	1.010	0.000	0.000	1.14700	0.09960
2	1.000	0.21700	0.12700	1.29924	0.53098
3	0.950	0.94200	0.19100	0.20000	0.15550
4	0.944	0.59488	0.06080	0.000	0.000
5	0.952	0.8236	0.01600	0.000	0.000
6	0.970	0.11200	0.07500	0.000	0.11149
7	0.946	0.000	0.09980	0.000	0.000
8	0.970	0.000	0.000	0.000	0.13306
9	0.933	0.29500	0.16600	0.000	0.000
10	0.933	0.07181	0.04628	0.000	0.000
11	0.947	0.03500	0.01800	0.000	0.000
12	0.952	0.06100	0.01600	0.000	0.000
13	0.945	0.13800	0.05800	0.000	0.000
14	0.918	0.14900	0.05000	0.000	0.000
<b>Total</b>		2.69804	0.92387	2.78897	1.03063

### IV CONCLUSION

In this paper, a scheme for representation of uncertain loads is presented and its handling in power system load flow studies has been shown. This can further be used for the voltage stability analysis and power system optimization studies. The critical case corresponding to uncertain inputs has been identified by the absolute value of minimum load flow Jacobian and corresponding load flow results are shown. Matlab2015 has been used to solve the formulated problem.

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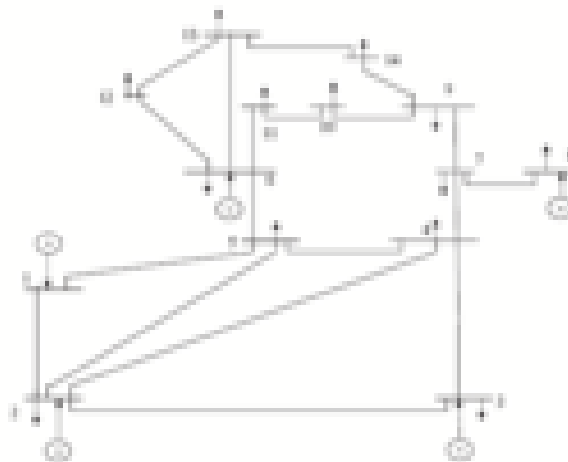
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### APPENDIX IEEE-14 BUS SYSTEM

IEEE 14 bus 20 lines system is shown in Figure 8 and its bus data are given in table 3. System Bus no. 1 is chosen as slack bus and bus no. 2, 3, 6, & 8 are generator buses. While the remaining buses 4, 5, 7, 9, 10, 11, 12, 13, & 14 are the load buses.



Bus No.	Vsp (pu)	P <sub>Li</sub> (MW)	Q <sub>Li</sub> (Mvar)
1	1.0600	0	0
2	1.0450	21.7000	12.7000
3	1.0100	94.2000	19.0000
4	1.0000	47.8000	-3.9000
5	1.0000	7.6000	1.6000
6	1.0700	11.2000	7.5000
7	1.0000	0	0
8	1.0900	0	0
9	1.0000	29.5000	16.6000
10	1.0000	9.0000	5.8000
11	1.0000	3.5000	1.8000
12	1.0000	6.1000	1.6000
13	1.0000	13.5000	5.8000
14	1.0000	14.9000	5.0000