RESONANT FREQUENCY AND EFFECTIVE RADIUS OF CIRCULAR SHAPE MICROSTRIP PATCH ANTENNA

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Abstract

In this paper the resonant frequency f_r and effective radius a_e are obtained in analytical form for a planner, circular shape microstrip patch antenna which is etched on a printed- circuit board so that the low profile antenna from the ground plane only by a thin layer of dielectric material. The formulas are found to have an error of less than 2.5 percent when compared with experimental data.

I. INTRODUCTION

In the last few years, wireless communication along with its various forms has become a part of everyday life. This dependency on wireless devices made it necessary to find antennas of small size antennas[1].Microstrip patch antennas are used in communication systems due to simplicity in structure, low manufacturing cost, small size and ease of installations[2][3].

II. CIRCULAR MICROSTRIP PATCH ANTENNA

Circular microstrip antenna consists of very thin metallic strip placed a small fraction of a wavelength above a ground plane[2].

Or, a circular microstrip antenna in its simplest form consists of a sandwich of two parallel conducting substrate. The lower conductor function as a ground plane and the upper conductor is a simple circular patch [5].

Referring to the dimensions of the circular patch, only one degree of freedom to control the radius, a of the patch. This would not change the order of the modes but the absolute value of the resonant frequency [2]-[3].

III. RESONANT FREQUENCY OF CIRCULAR MICROSTRIP ANTENNA

The modes that are supported by a circular microstrip antenna whose substrate height is small (h<<λ) are TM^z where z is taken perpendicular to the circular patch. For TM^z we need to first find the magnetic vector potential A_Z, which must satisfy in cylindrical coordinates, the homogeneous wave equation [2]

$$\nabla^2 A_z(\rho,\emptyset,z) + k^2 A_z(\rho,\emptyset,z) = 0 \quad (1)$$

Where are cylindrical coordinates of a point on the circular disk and $K^2 = \omega^2\mu\varepsilon$

For TMZ modes the electric and magnetic fields are related to the vector potential AZ by[2].

$$E_\rho = -j\frac{1}{\omega\mu\varepsilon}\frac{\partial^2 Az}{\partial\rho\,\partial z} H_\rho = \frac{1}{\mu}\frac{1}{\rho}\frac{\partial Az}{\partial\emptyset}$$
$$E_\emptyset = -j\frac{1}{\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial^2 Az}{\partial\emptyset\,\partial z} H_\emptyset = -\frac{1}{\mu}\frac{\partial Az}{\partial\rho}$$
$$E_z = -j\frac{1}{\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial Z^2} + k^2\right) A_Z H_Z = 0 \quad (2)$$

Subject to the boundary conditions of

$$E_\rho \le (0 \le \rho \le a, 0 \le \emptyset \le 2\pi, z = 0) = 0 \quad (3)$$
$$E_\rho \le (0 \le \rho \le a, 0 \le \emptyset \le 2\pi, z = h) = 0 \quad (4)$$
$$H_\emptyset \le (\rho = a, 0 \le \emptyset \le 2\pi, 0 \le z \le h) = 0s \quad (5)$$

Where ais the radius of circular patch and h is thickness of substrate.

The solution of equation (1) is given by

$$A_Z = E_{mnp} J_m(k_p\,\rho)[C\ \cos(m\emptyset) + D\ \sin(m\emptyset)[A\cos(k_z z) + B\sin(k_z z)] \quad (6)$$

Where E_{mnp}, A,B,C,D are constants and $J_m(x)$ are Bessel function of first kind of order m

Using equation (2) and boundary conditions given by equations (3), (4) and (5) we get

C≠0, B=0 (7)

$$k_z h = p\pi \ while\ A \neq 0 \quad (8)$$

$J'(k_\rho a) = 0$and D=0, which gives

$$k_\rho a = \square_{mn}; \quad (9)$$

Where $_{mn}$represents the zeroes of the derivative of the Bessel function $J_m(x)$

Now equation (11) reduces to

$$A_Z = E_{mnp} J_m(k_p \rho) C \cos(m\emptyset) A \cos(k_z z) \qquad (10)$$

With the constraint equation of

$$k^2{}_p + k^2{}_z = k^2 = \omega^2 \mu\varepsilon \qquad (11)$$

$$k_p = \frac{mn}{a} \qquad (12)$$

$$k_z = \frac{p\pi}{h} \qquad (13)$$

m=0,1,2,3……………….n=1,2,3……….and p=0,1,2,3…………….

The resonant frequencies of microstrip antenna are found using equation (11) to (13). Since for most typical microstrip antennas the substrate height h is very small, the fields along z are essentially constant and are presented by p=0 which gives k_z =0. Therefore the resonant frequencies for $TM^z{}_{mn0}$ can be written using equation (11) as [2]-[4]

$$(f_r)_{mno} = \frac{1}{2\pi\sqrt{\mu \in}} \frac{mn}{a} \qquad (14)$$

The dominant mode is the $TM^z{}_{110}$ whose resonant frequency is

$$(f_r)_{110} = \frac{1}{2\pi\sqrt{\mu \in}} \frac{mn}{a}$$

The value of $_{mn}$ =1.8412 so resonant frequency is given by

$$(f_r)_{110} = \frac{1.8412}{2\pi a\sqrt{\mu\varepsilon}} = \frac{1.8412\vartheta_0}{2\pi a\sqrt{\epsilon_r}} \qquad (15)$$

Where a is radius of circular patch ϵ_r is dielectric constant of substrate ϑ_0 speed of light in free space.

IV. EFFECTIVE RADIUS OF CIRCULAR MICROSTRIP ANTENNA

Due to the fringing fields between the patch and the ground plane, the effective dimensions of the antenna are greater than the actual dimensions. The fringing effect was larger due to the fact that some of the waves travel in the substrate and some in the air[2]-[3]. The above equation (15) derived for resonant frequency does not take into account fringing So, for the circular patch a correction is introduced byusing an effective radius ae to replace the actual radius a

The expression for effective radius a_e can be obtained as [2]-[4]

The zeroth-order resonant frequency of circular microstrip antenna for $TM^z{}_{110}$ mode is given by eq. (15)

$$f^{(0)} = \frac{11.8412}{2\pi a\sqrt{\mu\varepsilon}} \qquad (16)$$

Where a is the radius of circular patch and μ and ε are the permeability and permittivity of the dielectric substrate.

The zeroth-order capacitance of the circular disk over a ground plane is

$$C^{(0)} = \frac{\pi a^2 \in}{h} \qquad (17)$$

Since the zeroth-order frequency is given by eq.(16)

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad (18)$$

The Zeroth-order inductance is given by

$$L^{(0)} = \frac{\mu h}{\pi \square^2{}_{11}} (19)s$$

Where $\square_{11} = 1.841$corresponds to the first-order capacitance is derivative of the Bessel function of order 1. A simple algebraic formula for the first-order capacitance is available when the dielectric substrate is replaced by air

$$C = C_0(1 + \Delta) \qquad (20)$$

Where

$$\Delta = \frac{2h}{\pi a}\left[\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right] for\ \varepsilon = \varepsilon_0 \qquad (21)$$

For ε different than ε_0 the capacitance is expressed by

$$C = \frac{\pi a^2 \varepsilon}{h}(1 - C_0 + C'_0 - C''_0 + \cdots) \qquad (22)$$

For small h/a ,C'_0,C''_0 can be neglected and

$$C_0 = \frac{-2\left(\frac{h}{a}\right)}{\left(\frac{\varepsilon}{\varepsilon_0}\right)}\left[\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right] \qquad (23)$$

Comparing the eq. (23) with eq. (21) suggests the following approximate formula for C_o

$$C_0 = -\Delta = \frac{-2h}{\pi\left(\frac{\varepsilon}{\varepsilon_0}\right)a}\left[\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right] \qquad (24)$$

The first-order resonant frequency f for TM^z_{110} mode is given by

$$f = \frac{1}{2\pi\sqrt{L^{(0)}C^{(0)}(1+\Delta)}} \quad (25)$$

.Now putting the value of $L^{(0)}, C^{(0)}$ and Δ from eq.(19),eq.(17) and eq.(24) in eq.(25) then we get

$$f = \frac{\square_{11}}{2\pi\sqrt{\mu\varepsilon a\sqrt{1} + \frac{2h}{\pi a\,\varepsilon_r}\left[\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right]}} \quad (26)$$

Let

$$a_e = a\sqrt{1} + \frac{2h}{\pi a \varepsilon_r}\left[\ln\left(\frac{\pi a}{2h}\right) + 1.7726\right] \quad (27)$$

This eq. (27) gives the expression of effective radius a_e
Now eq.(26) reduces to

$$f = \frac{1.8412}{2\pi a_e\sqrt{\mu\varepsilon}} = \frac{1.8412\vartheta_0}{2\pi a_e\sqrt{\varepsilon_r}} \quad (28)$$

Where ε_r is dielectric constant of substrate and ϑ_0 is speed of light in free space.

V. CONCLUSION

Thus using cavity model for analysis, the expression of resonant frequency is obtained. The expression of effective radius a_e is also resonant frequency it is found that for circular patch there is only one degree of freedom to control which is radius of circular patch. This does not change the order of the modes but it does change the absolute value of the resonant frequency.

VI. REFERENCES

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