Queuing System with Encouraged Arrivals, Impatient Customers and Retention of Impatient Customers for Designing Effective Business Strategies

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Abstract – In todays' scenario of uncertain business environment and fierce competition, organisations don't leave any stone unturned to stay ahead of others. They often introduce lucrative offers and discounts to attract customers. These encouraged customers at times result is heavy rush and waiting time of customers at the service facility increases. Long waiting time results in customer impatience and customers after waiting for more than their threshold waiting time limit abandon the facility and renege. Reneging is a loss to business and it is very important for an organisation to design strategies in advance to retain the reneging customers. It can be done effectively if the performance of the system can be measured in advance with some probabilities. In this paper a single-server queuing model is developed to analyse the system encountering above mentioned challenges. The model is then solved iteratively and various performance measures are derived. Numerical illustrations and economic analysis are presented. Sensitivity analysis of the model is also performed to study the impact of various parameters on cost model functions.

Key words: Queuing Theory, Customer Impatience, Reneging, Encouraged Arrivals, Retention

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I. INTRODUCTION AND LITERATURE SURVEY

Managing business in today's volatile and uncertain environment is a challenging task. Organisations are facing fierce competition from not only domestic but also from international markets. Any product from any corner of the world is available at the fingertips of the customers because of technological advancements. In order to fight this tough competition and stay ahead organisations introduce various lucrative deals and discounts to attract customers. These attracted customers are termed as encouraged arrivals (Som and Seth, 2017) (Som and Seth, 2017) developed queuing model with encouraged arrivals and impatient customers for managing the business effectively under uncertain business environment. And presented its economic analysis as well. This paper is a further extension where retention of impatient customers in case of encouraged arrivals is presented. The phenomenon of encouraged arrivals can also be understood as contrary to discouraged arrivals discussed (Kumar and Sharma, 2014). They mentioned that customers get discouraged to join once they look in to large system size (Reynolds, 1968) presented multi-server queuing model with discouragement (Natvig, 1975). studied single server queuing model with discouraged arrivals with state dependent parameters. As encouraged arrivals results in large customer base, due to limited service rate waiting time of the customers in the system increases and as a result length of the queue increases. Due to longer queues many customers standing in the queues may get impatient and start leaving the queue without completion of their service. This is termed as reneging in queuing literature.The concept of reneging appears in queuing theory in the work of (Barrer, 1957) (Haight, 1959) and (Ancker and Gafarian, 1963) (Ancker and Gafarian, 1963) After that numbers of authors have studied the concept of reneging. (Wang, et. al., 2010) presented an extensive review on queuing systems with impatient customers. Reneging is a loss to business and it hampers the goodwill of the organisation. Hence there is a need to design strategies in advance to retain customers leaving the system without completion of service. The concept of retention was introduced (Kumar and Sharma, 2012) and (Kumar and Sharma, 2013). They studied retention of impatient customers in single server as well as multi-server case.

In this paper we develop a single-server Markovian queuing system with customer impatience and retention of impatient customers. The paper is organized as per following details. Mathematical model formulation is presented in section 2 while section 3 presents steady-state solution of the model. Section 4 deals with measures of performances. Numerical illustration is presented in section 5. Section 6 deals with economic analysis and sensitivity analysis of the model. Conclusion and future scope are given in section 7.

II. MATHEMATICAL MODEL FORMULATION

A single-server queuing model is formulated under following assumptions:

(i) The arrivals occur one by one in accordance to Poisson process with parameter $\lambda(1+\eta)$, where 'η' represents the percentage increase in number of customers calculated from past or observed data.

(ii) Service times are exponentially distributed with parameter μ.

(iii) Customers are serviced in the order of their arrival i.e. first come first served.

(iv) Service is provided through a single channel.

(v) The capacity of the system is finite say, N.

(vi) Reneging times are exponentially distributed with parameter ξ.

(vii) The probability of retention of a reneged customer is q and the probability that customer is not retained is p = 1−q.

In steady state, equations governing the model are given by:

$$0 = -\lambda(1+\eta)P_0 + \mu P_1 \quad (1)$$

$$0 = \lambda(1+\eta)P_{n-1} + \{-\lambda(1+\eta) - \mu - (n-1)\xi p\}P_n + (\mu + n\xi p)P_{n+1} \quad (2)$$

$$0 = \lambda(1+\eta)P_{N-1} - \{\mu + (N-1)\xi p\}P_N \quad (3)$$

III. STEADY-STATE SOLUTION

On solving (1) - (3) iteratively we get;

$$P_n = \Pr\{'n'\text{customers in the system}\} = \prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0,\ 1 \le n \le N-1 \quad (4)$$

And the probability that system is full is given by:

$$P_N = \Pr\{\text{sytem is full}\} = \prod_{i=0}^{N-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0 \quad (5)$$

Using condition of normality $\sum_{n=0}^{N} P_n = 1$

$$P_0 = \Pr\{\text{system is empty}\} = \left\{1 + \sum_{n=1}^{N}\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}\right\}^{-1} \quad (6)$$

IV. MEASURES OF PERFORMANCE

1. Expected System Size (L_s)

$$L_s = \sum_{n=0}^{N} nP_n = \sum_{n=0}^{N} n\left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0\right\} \quad (7)$$

2. Expected queue length (L_q)

$$L_q = \sum_{n=0}^{N}(n-1)P_n = \sum_{n=0}^{N}(n-1)\left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0\right\} \quad (8)$$

3. Average rate of reneging (R_r)

$$R_r = \sum_{n=1}^{N}(n-1)\xi p P_n = \sum_{n=1}^{N}(n-1)\xi p\left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0\right\} \quad (9)$$

4. Average rate of retention (R_R)

$$R_r = \sum_{n=1}^{N}(n-1)\xi q P_n = \sum_{n=1}^{N}(n-1)\xi q\left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0\right\} \quad (10)$$

V. NUMERICAL ILLUSTRATION

In this section we present numerical illustration of the above model.

Variation in L_s, L_q, R_r and R_R with respect to λ

We take, $N = 10, \mu = 3, \xi = 0.2, p = 0.4,\ \eta = 0.5$

Table -1

Average rate of arrival (λ)	Expected System Size (Ls)	Expected Queue Length (L_q)	Average rate of Reneging (R_r)	Average rate of Retention (R_R)
2	4.004591	3.12906	0.250325	0.375487
2.2	4.850677	3.93233	0.314586	0.471880
2.4	5.637055	4.688786	0.375103	0.562654
2.6	6.321101	5.353144	0.428252	0.642377
2.8	6.889811	5.909442	0.472755	0.709133
3	7.350464	6.362471	0.508998	0.763497
3.2	7.719431	6.726812	0.538145	0.807217
3.4	8.014679	7.019259	0.561541	0.842311
3.6	8.252229	7.255106	0.580408	0.870613
3.8	8.445101	7.446932	0.595755	0.893632
4	8.603399	7.604582	0.608367	0.912550
4.2	8.734808	7.735583	0.618847	0.928270
4.4	8.845127	7.845641	0.627651	0.941477
4.6	8.938737	7.939084	0.635127	0.952690
4.8	9.018968	8.019204	0.641536	0.962304
5	9.088368	8.088531	0.647083	0.970624
5.2	9.148909	8.149023	0.651922	0.977883

It can be observed that with increase in arrival rate the expected length of the system increases and so as expected queue length, while increase in reneging rate means that with increase in number of customers waiting time increases and customers leave the system and increase in rate of retention also shows that by employing retention strategies many customers can be retained. Similarly by varying service rate, the numerical results are obtained below:

Table -2

Variation in L_s, L_q, R_r and R_R with respect to μ

We take, $N = 10, \lambda = 3, \xi = 0.2, p = 0.4\ \eta = 0.5$

Average rate of service (μ)	Expected System Size (Ls)	Expected Queue Length (L_q)	Average rate of Reneging (R_r)	Average rate of Retention (R_R)
3	7.350464	6.362471	0.508998	0.763497
3.1	7.16726	6.182199	0.494576	0.741864
3.2	6.976688	5.995061	0.479605	0.719407
3.3	6.779503	5.801846	0.464148	0.696222
3.4	6.576605	5.603487	0.448279	0.672418
3.5	6.369025	5.401036	0.432083	0.648124
3.6	6.157896	5.195639	0.415651	0.623477
3.7	5.944422	4.988502	0.39908	0.598620
3.8	5.729844	4.780859	0.382469	0.573703
3.9	5.515401	4.573936	0.365915	0.548872
4	5.302303	4.368914	0.349513	0.524270
4.1	5.091692	4.166902	0.333352	0.500028
4.2	4.884622	3.968913	0.317513	0.476270
4.3	4.682034	3.775842	0.302067	0.453101
4.4	4.484744	3.588456	0.287077	0.430615
4.5	4.293436	3.407388	0.272591	0.408887
4.6	4.108659	3.233133	0.258651	0.387976

It can be observed that with increase in service rate the expected length of the system decreases and so as expected queue length, and due to less waiting time with fast service, reneging rate also decreases and hence rate of retention also decreases.

VI. ECONOMIC ANALYSIS

In this section economic analysis of the modelis performed by developing the functions of total expected cost (TEC), total expected revenue (TER) and total expected profit (TEP).

Total expected cost (TEC) is given by:

$$TEC = C_s\mu + C_h \sum_{n=0}^{N} n\left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0\right\} + C_r\sum_{n=1}^{N}(n-1)\xi p\left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0\right\} + C_R\sum_{n=1}^{N}(n-1)\xi q\left\{\prod_{i=0}^{n-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0\right\} + C_L\lambda\prod_{i=0}^{N-1}\frac{\lambda(1+\eta)}{\mu+i\xi p}P_0$$

Total expected revenue (TER) is given by:

$$TER = R \times \mu \times (1 - P_0)$$

$$TER = R \times \mu \times \left[1 - \left\{1 + \sum_{n=1}^{N} \prod_{i=0}^{n-1} \frac{\lambda(1+\eta)}{\mu + i\xi p}\right\}^{-1}\right]$$

Total expected profit (TEP) is given by:

$$TEP = TER - TEC$$

Where, C_s= Service cost per unit time, C_h= holding cost per unit per unit, C_r= Cost per reneging unit per unit time, C_L= Cost per lost unit per unit time, R= Revenue earned per unit per unit time

The cost model formulated above if translated in MS EXCEL and sensitivity analysis is performed for varying rates of arrival and service.

Table -3

Variation in $TEC, TER \text{ and } TEP$ with respect to λ

Taking,
$N = 10, \mu = 3, p = 0.4, \xi = 0.2, \ \eta = 0.5, C_S = 10, C_L = 15, C_h = 2, C_r = 2, R = 200$

Average rate of arrival (λ)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
2	39.74067	525.3182	485.5775
2.2	42.63425	551.0083	508.374
2.4	45.82523	568.9611	523.1358
2.6	49.17749	580.7739	531.5964
2.8	52.58652	588.2211	535.6346
3	55.98916	592.7958	536.8066
3.2	59.35513	595.5715	536.2164
3.4	62.67443	597.252	534.5775
3.6	65.94749	598.274	532.3265
3.8	69.17911	598.9011	529.722
4	72.37535	599.2904	526.915
4.2	75.54208	599.5352	523.9931
4.4	78.68448	599.6912	521.0067
4.6	81.80686	599.792	517.9852
4.8	84.91274	599.8581	514.9453
5	88.00498	599.9019	511.897
5.2	91.08587	599.9314	508.8455

It can be observed that the total expected profit increases with increase in average rate of arrival, but starts falling down after reaching a maximum value. It is due to the fact that with fixed service rate, after certain level with increasing load on service, cost increases rapidly than revenue, owing to longer queues and increasing reneging.

Table -4

Variation in $TEC, TER \text{ and } TEP$ with respect to μ

We take, $N = 10, \lambda = 3, \xi = 0.2, \ \eta = 0.5, \ p = 0.4, C_S = 10, C_L = 15, C_h = 2, C_r = 2, R = 200$

Average rate of service (μ)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
3	55.98916	592.7958	536.8066
3.1	55.84102	610.7378	554.8967
3.2	55.70445	628.2417	572.5373
3.3	55.58315	645.2534	589.6703
3.4	55.48097	661.7202	606.2392
3.5	55.40176	677.5925	622.1907
3.6	55.3493	692.8256	637.4763
3.7	55.32715	707.3811	652.054
3.8	55.33854	721.2278	665.8893
3.9	55.38634	734.3428	678.9564
4	55.47293	746.7115	691.2386
4.1	55.60015	758.3283	702.7282
4.2	55.76934	769.196	713.4266
4.3	55.98128	779.325	723.3437
4.4	56.23623	788.7328	732.4966
4.5	56.53399	797.4431	740.9091
4.6	56.8739	805.4843	748.6104

It can be observed that the firms profit keeps on increasing with an increasing rate of service due to reducing reneging rate.

Table -5

Variation in TEC, TER and TEP with respect to q, the probability of retaining a customer.

We take,

$$N = 10, \lambda = 2, \mu = 3, \ \eta = 0.5, \ \xi = 0.4, C_s = 10, C_L = 15, C_h = 2, C_r = 2, R = 200$$

Probability of retaining a customer (q)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
0.1	36.1774	479.4618	443.2844
0.15	36.28892	481.8394	445.5505
0.2	36.4114	484.317	447.9056
0.25	36.54641	486.9021	450.3557
0.3	36.69579	489.6031	452.9073
0.35	36.86174	492.429	455.5672
0.4	37.04684	495.3896	458.3428
0.45	37.25418	498.4957	461.2415
0.5	37.48741	501.7586	464.2712
0.55	37.75089	505.1904	467.4395
0.6	38.04979	508.8037	470.7539
0.65	38.39027	512.6112	474.2209
0.7	38.77961	516.6254	477.8458
0.75	39.2264	520.858	481.6316
0.8	39.74067	525.3182	485.5775
0.85	40.33403	530.012	489.6779
0.9	41.01964	534.9397	493.9201

It can be observed that with increase in the probability of retaining a customer which can be done by employing effective retention strategies, profit of the firm keeps on increasing as customer choose to wait and complete their service than leaving the system without completion of service.

VIII. CONCLUSION AND FUTURE SCOPE

The results of the paper could be of immense use for any organization encountering the phenomenon of encouraged arrivals, customers getting impatient and leaving the system and where organisation needs to design retention strategies to retain/ renege customers. By knowing the measures of performance in advance, effective strategies could be designed for smooth administration. By adopting and implementing this model the financial aspects of the system could also be examined.

Further, model could be studied with infinite capacity system and multi-server model could be developed. Model could also be studied in transient state and optimization of service rate and arrival rate in the model could be achieved.

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