MATHEMATICAL MODEL FOR THE FLOW STRESS CONSTITUTIVE EQUATION USING SERIES EXPANSION

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Abstract-*Conventional constitutive equations defining the flow stress relationship with the allied parameters are independent of one or more parameters. So the combined effect of all the flow parameters on true stress cannot be imagined in the existing frame of reference. In addition the same set of arbitrary constants does not works for all true strain, strain and temperature values for such models and their equations, which further adds new drawbacks while working with these relations. The present article works to complete the relationship between such flow parameters and the equation has been validated to Al 2024 aluminum alloy.*

Keywords- *Material Modeling, Hot deformation, Flow/ True stress, True strain rate deformation, Thermo-mechanical process.*

I INTRODUCTION

Deformation Constitutive equations are those which describes the linear and non-linear relationship among the process variables viz. effective stress, effective strain, effective strain rate and temperature at different deformation levels. Such equations are required for the development of realistic dynamic material models involving various processes. The functional form of the constitutive relation representing flow behavior is given by

$$\sigma = f(\varepsilon, \dot{\varepsilon}, T) \qquad (1)$$

Where, σ represents the Flow stress, ε represents the True strain, $\dot{\varepsilon}$ represents the True strain rate and T represents the temperature.

The prominent material models in this category can be divided to three categories;

(a) First stage equations:

i) Holloman equation [1, 2]

It gives a direct dependency of flow stress on strain. According to it the flow stress can be defined in terms of strain as,

$$\sigma = k\varepsilon^n \qquad (2)$$

Where, k and n are the strength coefficient and strain hardening exponent respectively.

ii) Ludwik equation [3]

This model includes the effect of yield stress σ_0. According to it flow stress can be expressed as,

$$\sigma = \sigma_0 + k\varepsilon^n \qquad (3)$$

iii) Swift equation [4]

This model includes the effect of yield strain ε_0.

$$\sigma = k\left(\varepsilon_0 + \varepsilon\right)^n \qquad (4)$$

iv) Voce equation [2, 4]

$$\sigma = A - Ke^{(-C\varepsilon)} \qquad (5)$$

Equations 1, 2, 3 and 4 do not have strain rate and temperature terms. Thus the effects of these parameters cannot be observed.

(b) Second stage equations:

Another type of equations containing strain terms are:

i) Strain independent power law [1]

It is,

$$\sigma = k_1\dot{\varepsilon}^m \qquad (6)$$

k_1 is constant for particular strain, strain rate and temperature, The exponent "m" is also constant at a given strain and temperature

ii) Strain dependent power law [5]

$$\sigma = k_1\varepsilon^n\dot{\varepsilon}^m \qquad (7)$$

In these equations the dependency of flow stress on strain and strain rate has been considered but the effect of temperature has been neglected.

(c) Third stage equations:

They are the equations that contain both strain rate and temperature in determining the flow stress. They are called Arrehenius equation, Exponential law, Hyperbolic sine law [6], [7]. They are often used as the kinectic models and generally referred to define such relationship. These equations are as follows in equation 2, 3, 4 respectively.

$$\dot{\varepsilon}\exp(Q/RT)=A\sigma^{n} \quad (8)$$

$$\dot{\varepsilon}\exp(Q/RT)=A`\exp(\beta\sigma) \quad (9)$$

$$\dot{\varepsilon}\exp(Q/RT)=A``(\sinh\alpha\sigma)^{n`} \quad (10)$$

A common drawback to these equation is the lack of absolute relationship between all the four variables i.e. true stress (σ), true strain *(ε)*, true strain rate ($\dot{\varepsilon}$) and deformation temperature (T) i.e. one or more parameter is absent in the constituting relationship. However the equation defined by "N.S. Babu, S. B. Tiwari and B. Nageshwara Rao" in the paper entitled "Modified in stability condition for identification of unstable metal flow regions in processing maps of magnesium alloys", involves the inclusion of all parameters [9]. Mathematically, Babu Model has the following functional form,

$$F=f(\varepsilon, \dot{\varepsilon}, T, d) \quad (11)$$

It obtains the constitutive relationship in logarithmic scale by using Taylor's series expansion defined as,

$$\ln\sigma = \sum_{i=1}^{I}\sum_{j+1}^{J}\sum_{k=1}^{K} C_{ijk}\varepsilon^{i-1}\Theta^{j-1}(\ln\dot{\varepsilon})^{k-1} \quad (12)$$

Where, Θ = 1000/T and T is in Kelvin.

This relation however involves true strain but the number of constants C_{ijk} need to completely define the relationship is 48 numbers which are very exhaustive with no physical significance. Another difficulty in this formula occurs during estimation of flow stress, since three values of dependent variable and 48 values of arbitrary constants has to be substituted for each theoretical calculation.

The present work is an attempt to remove these drawbacks by the inclusion of true strain (ε) parameter to the remaining three parameter with limited number of constants. In order to validate the equation, the data were taken from "HOT WORKING GUIDE" [9].

II PROPOSED MODEL & ITS VALIDATION

This article shows a constitutive equation with comparatively lesser number of coefficients in which different parameters and arbitrary constants follow the following form,

$$\ln\sigma = \sum_{i=1,2}\sum_{j=i}^{i+1} Aij\varepsilon^{i-1}X^{j-1} \quad (13)$$

Or, $$\sigma = exp\left(\sum_{i=1,2}\sum_{j=i}^{i+1} Aij\varepsilon^{i-1}X^{j-1}\right) \quad (14)$$

Where, X= ln $\dot{\varepsilon}$, A_{ij} are the functions of T.

The alloy taken under study is Al-2024 Aluminum alloy. *ln*(True stress)-*ln*(True strain rate plots has been shown in fig-1 at ε = 0.2, 0.4 which helps to establish σ-ε relationship.

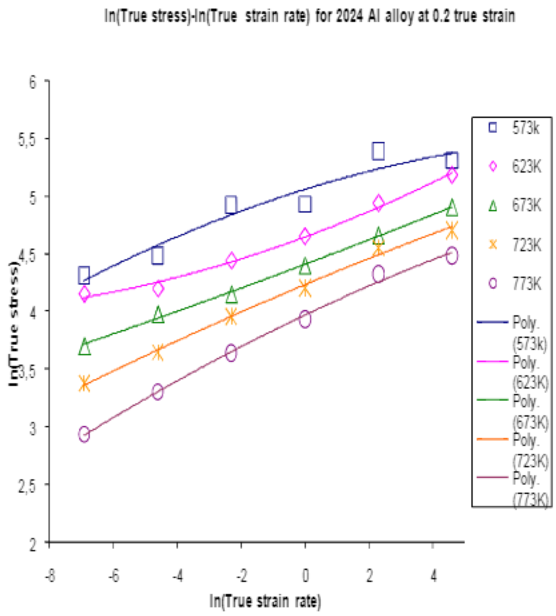

Figure 1: (a) σ-ε relationship at (a)ε = 0.2 (b) ε = 0.4

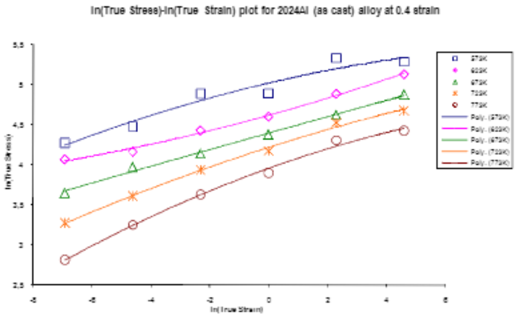

Figure1:(b) σ-ε relationship at (b) $\varepsilon = 0.4$

Similar relation plots were made w.r.t other parameters.

III RESULTS & DISCUSSION

Proceeding through the equation (14), we obtain the value of parameter for our case study alloy Al 2024. They are shown in Table-1. A Comparison between the experimental results (R) & calculated results (c) by equation (14) at different temperatures w.r.t. other parameters are made in terms of the flow stress curves in fig-2.

A_{ij}	*Value of* A_{ij}
A_{11}	2367.5/T +0.921
A12	0.0002T-0.0249
A_{13}	$-112420/T^2 +338.5/T -0.2513$
A_{21}	-275.5/T +0.3235
A_{22}	-0.026
A_{23}	$9620/ T^2 -5.4/T-0.0195$

Table 1: *Value of* A_{ij}

These plots made in Fig-2 clearly show that the present model can predict the value of flow stress over a wide range of practically feasible parameters. Further it reduces the number of constants from 48 to 13 in number.

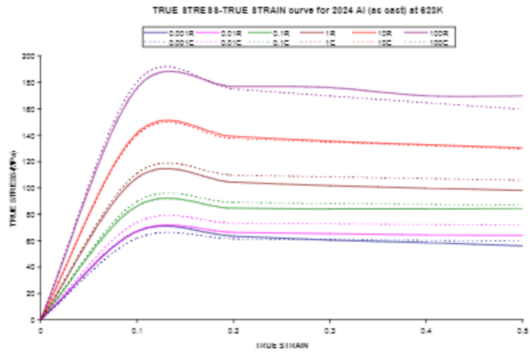

Figure2: (a)

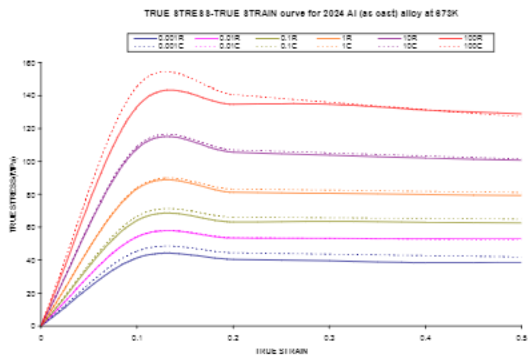

Figure2: (b)

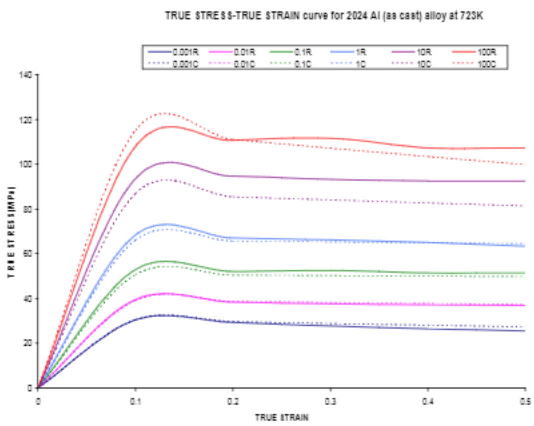

Figure2: (c)

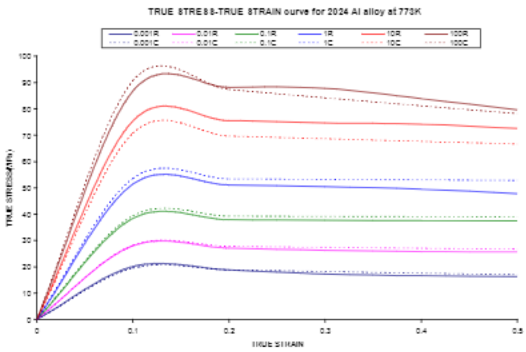

Figure-2: (d) Comparison between the experimental results-R(full lines) & calculated results-C (Dotted lines) by equation (14) at (a) 623K (b) 673K (c) 723K (d) 773K

IV CONCLUSION

Analysis carried out in this research article reflects the dependency of the flow stress on other deformation parameters. The main conclusions are as follows,

(a) The proposed model shows a good correlation between experimental and theoretical results.

(b) It shows a direct relation of flow stress with its influencing factors viz. strain, strain rate and temperature.

(c) It reduces the number of constants from 48 in N.S. Babu Model [9] to 13 in number.

(d) Values at stringent experimental conditions can be obtained by Extrapolation/ interpolation.

(e) This model may be used to eliminate the experimental error due to sudden change in physical conditions like voltage fluctuation.

(f) The proposed model can ably covers a wider range of materials exhibiting the similar compressive behavior under the deforming compressive load and ably fingerprints the experimental results with accuracy.

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