A TWO STAGE TANDEM REPAIRABLE REMANUFACTURING SYSTEM WITH WORK-IN-PROCESS (WIP) INVENTORY CONTROL

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Abstract-*This paper deals with an unreliable remanufacturing system involved in the processing and remanufacturing of WIP inventory. While processing the items to be recycled, the system may become out of order followed by a two stage tandem repair. The system capacity K is assumed to be finite. As soon as the items start entering the system and queue up to seek service, the server takes some start up time to activate and then waits to serve until the N items of WIP inventory are accumulated. Once the system becomes full, further arrival of items is not allowed until the queue length decreases to a pre-specified level F. The time dependent behavior of the system is analyzed using Runge-Kutta 4^{th} order method. To explore the system's effectiveness, the model formulated has been tested numerically.*

Keywords*-Unreliable remanufacturing system, WIP inventory, Tandem repair*

I INTRODUCTION

Work in process (acronym: WIP) or **in-process inventory** includes the set of unfinished items or used items for products in a production process. These items are those which are not yet manufactured as desired but either are just being fabricated or waiting in a queue for further processing or lie in a buffer storage. Remanufacturing differs from other recovery processes in its completeness: a remanufactured product should match the same customer expectation as new products. Remanufacturing is also described as an industrial process in which worn-out products or the used products are restored equivalent to new condition. Through an industrial process in a factory environment, a discarded product is completely disassembled. Useable parts are cleaned, refurbished, and put into inventory. Then the product is reassembled from the old parts (and where necessary, new parts) to produce a unit fully equivalent and sometimes superior in performance and with expected lifetime to the original new product. emanufacturing helps in cost cutting of the entire system as reworked units cost lesser than the new units. Many esearchers have worked on WIP inventory control in context of manufacturing sytems. Tsiotras and Tapiero (1992) studied the mutual effects of WIP inventory, manufacturing process reliability and quality control in a queue like manufacturing system. Nye et al. (2001) developed a model based on queueing theory, which permits it to estimate WIP levels as a function of decision variables, batch size and setup time. Papadopoulos and Vidalis (2001) worked on the optimal buffer allocation (OBA) to minimize the average work in process (WIP) inventory, subject to minimum required throughput. Ma and Koren (2004) introduced a novel approach to meet the production target and to minimize the work in process inventory for large manufacturing systems with buffers. Qiu (2005) introduced a practical solution to a manageable and well distributed WIP control system by addressing issues such as real time performance, scalability and reconfigurability. Chan et al. (2007) established a mathematical model for the production control problem of a manufacturing system with time delay, demand uncertainty and extra capacity with the objective of minimizing the mean costs for WIP inventory and occupation of extra production capacity. Liu and Lian (2009) considered the cost effective inventory control of work-in-process (WIP) and finished products in a two stage distributed system. Ahiska and King (2010) discussed the inventory optimization of a single product recoverable manufacturing system where customer demands were satisfied through either regular production of new items or remanufacturing of returned items. In a queueing system or an inventory system, the server is always susceptible to random failure and thus considered unreliable. The efficiency of the entire system becomes low due to unreliable server. The broken down server is repairable and can be restored to make it available to serve the queued up units. Many researchers have contributed in the direction related to unreliable systems. Dohi et al. (2001) developed a stochastic model for the optimal control of preventive maintenance schedule and safety stocks in an unreliable manufacturing environment. An unreliable production-inventory model with two phase erlang demand arrival process was studied by Wang et al. (2002). Wang and Srinivasan (2005) investigated a production-inventory system with hyper-exponential renewal demand processes. They obtained steady state distribution for the stability condition of the inventory process and used it to calculate system's performance. Ke(2006) studied the control policies of an M/G/1 queueing system with a startup and unreliable server, in which the length of the vacation period was controlled either by the number of arrivals during the idle period or by a timer. An analytical model was formulated by Kenne et al. (2007) for the joint determination of an optimal age-dependent buffer inventory and preventive maintenance policy in a production environment that was subject to random machine breakdowns. The system characteristics of a two-unit repairable system were studied by Ke et al. (2008) from a Bayesian viewpoint with different types of priors assumed for unknown parameters, in which the service station is unreliable. Choudhury and Tadj (2009) studied the steady state behavior of an M/G/1 queue with an additional second phase of optional service subject to random breakdowns at any instant while serving the customers and delayed repair. Hajji et al. (2011) considered joint production control and product specifications decision making in a failure prone manufacturing system.

To avoid the halt of the manufacturing process due to sudden failure of the system, the organization has to be equipped with skilled repair facility. The repair of the inoperative system can be done in phases depending on the nature of the breakdown; whether it is minor or major. Ching (2001) considered the manufacturing systems of m identical unreliable machines producing one type of product. The operating time of each machine is assumed to be exponentially distributed and the repairing process of a machine required more than one phase. Ke and Lin (2008) modeled a manufacturing system consisting of a number operating and spare machines under the supervision of a group of technicians in a repair facility. The machine failure was considered in Poisson fashion followed by a two phase tandem repair. A repairable queueing model with a two-phase service in succession, provided by a single server, was investigated by Dimitriou and Langaris (2010). In today's manufacturing environment, the system designer is much concerned about the incorporation of the policies which prove to be useful in reducing the idle time of the server and using it more efficiently. One of such policies is N-policy. It refers to the accumulation of N units in the queue to initiate the server to start the service. Various researchers have implemented the concept of N-policy while modelling the manufacturing systems mathematically. Medhi and Templeton (1992) studied a Poisson input queue under N-policy and with a general start up time. A bulk input queueing system with batch gated service and multiple vacation policy was presented by Bacot and Dshalalow (2001). Krishnamoorthy and Deepak (2002) incorporated N-policy for an M/G/1 queue. Wang et al. (2005) performed maximum entropy analysis of the N-policy M/G/1 queueing system with server breakdowns and general startup times. A bulk quorum queueing system with N-policy and Bernoulli vacation schedule was suggested by Tadj et al. (2006). The maximum entropy approach for batch-arrival queue under N-policy with an unreliable server and single vacation was employed by Ke and Lin (2008). An M/G/1 retrial G-queue with preemptive resume and feedback under N-policy vacation subject to the server breakdowns and repairs was investigated by Liu et al. (2009). A near optimal buffer allocation plan (NOBAP) was developed by Aksoy and Gupta (2010) specifically for a cellular remanufacturing system with finite buffers where the servers follow N-policy.

In order to maintain the smooth functioning of the manufacturing system the arrivals in a queue should be controlled. This can be done by implementing F-policy by the system designer. In WIP inventory, F-policy may be employed to control the arrival of the material to be remanufactured. Wang et al. (2008) studied the optimal control of a finite capacity G/M/1 queueing system combined with the F-policy (for controlling arrival to a queueing system) and an exponential startup time before start allowing customers in the system. Wang and Yang (2009) presented the control policy of a removable and unreliable server for an M/M/1/K queueing system, where the removable server operates on F-policy. The so-called F-policy means that when the number of customers in the system reaches its capacity K (i.e. the system becomes full), the system will not accept any incoming customers until the queue length decreases to a certain threshold value F. Yang et al. (2010) analyzed the F-policy M/M/1/K queueing system with working vacation and an exponential startup time. The F-policy is used to deal with the issue of controlling arrivals to the queueing system. The startup time of the server before allowing customers to enter the system is also considered.In this study we have examined the time dependent performance of a remanufacturing system of WIP inventory working with (N, F) policy. The system is of finite capacity and is unreliable. The failed system can be repaired in two phases. The rest of the paper is organized as follows. In the next section 2, the model is justified by a practical example. Section 3 provides the requisite assumptions and notation for the mathematical formulation of the model. The transient equations governing the model have been formulated using the transition diagram in section 4.

These equations have been solved using Runge-Kutta 4^{th} order method to determine transient probabilities and using these probabilities some important performance measures have been evaluated as discussed in section 5. Section 6 is devoted the numerical illustrations. Finally the conclusion is provided in section 7.

II PRACTICAL JUSTIFICATION OF THE MODEL

Consider a PET (Polyethylene Terephthalate) plastic recycling plant. PET is used as a raw material for making packaging materials such as bottles and containers for packaging a wide range of food products and other consumer goods. Examples include soft drinks, alcoholic beverages, detergents, cosmetics, pharmaceutical products and edible oils. PET is one of the most common consumer plastics used. The empty PET packaging is discarded by the consumer after use and becomes PET waste.

In the recycling industry, this is referred to as "post-consumer PET." The post consumer PET units are brought for recycling, which can be said to be WIP inventory here. The units to be recycled are queued up before getting entry into the treatment plant whose capacity is finite (say K). Once the N- units are accumulated for recycling these are sent to the plant. Firstly, the treatment process is done which includes crushing, washing, separating and drying. Recycling plant will further treat the post-consumer PET by shredding the material into small fragments. These fragments still contain residues of the original content, shredded paper labels and plastic caps. These are removed by different processes, resulting in pure PET fragments, or "PET flakes". PET flakes are used as the raw material for a range of products which include polyester fibers (a base material for the production of clothing, pillows, carpets, etc.), polyester sheets, strapping, or back into PET bottles etc. Once the plant becomes full, further arrival is not allowed till the queue length reduces to a pre-specified level F. The recycling plant is unreliable and may face random breakdowns. The plant is repairable and can be repaired in two phases depending on the intensity of the fault. The following figure portrays the mechanism of recycling plant.

III ASSUMPTIONS AND NOTATIONS

We consider a remanufacturing system involved in the recycling of the used commodities to produce the new one. The WIP inventory arrives at the remanufacturing system for recycling in Poisson fashion with rate λ. On the accumulation of N units in the system, the server starts providing the service with the rate $\propto$. The system capacity is finite (K). Once the system is full no arrival is allowed till the queue length decreases to a pre-specified level F. Other notations associated with the model are as follows:

- γ : start up time of the repairman
- α : breakdown rate of the server
- β : repair rate of the server
- $(1-\delta)$: probability that the server is restored after 1^{st} phase repair
- δ : probability that the broken down server moves to 2^{nd} phase repair
- $P_{0,0}(t)$: probability that the system is empty and the server is idle
- $P_{0,n}(t)$: probability that there are n units in the system and server is busy
- $P_{0,k}(t)$: probability that the system is full and server is busy
- $P_{1,0}(t)$: probability that the server takes start up and no unit is present in the system
- $P_{1,n}(t)$: probability that the server takes start up and n unit are present in the system
- $P_{2,0}(t)$: probability that the server is in broken down state and no unit is present in the system
- $P_{2,n}(t)$: probability that the server is in broken down state and n units are present in the system
- $P_{3,0}(t)$: probability that the server is under 2^{nd} phase repair and no unit is present in the system
- $P_{3,n}(t)$: probability that the server is under 2^{nd} phase repair and n units are present in the system

IV THE GOVERNING EQUATIONS

The equations governing the model are constructed with the help of transition diagram which are as follows:

$$\frac{d}{dt}P_{0,0}(t) = -\gamma P_{0,0}(t) + \propto P_{0,1}(t) \qquad (1)$$

$$\frac{d}{dt}P_{0,n}(t) = -(\gamma + \propto)P_{0,n}(t) + \propto P_{0,n+1}(t) \quad 1 \le n \le F \qquad (2)$$

$$\frac{d}{dt}P_{0,n}(t) = -\propto P_{0,n}(t) + \propto P_{0,n+1}(t)$$
$$F+1 \le n \le K-1 \qquad (3)$$

$$\frac{d}{dt}P_{1,0}(t) = -(\lambda + \alpha)P_{1,0}(t) + \beta P_{3,0}(t) + (1-\delta)\beta P_{2,0}(t) + \gamma P_{0,0}(t)$$

$$\frac{d}{dt}P_{0,K}(t) = -\propto P_{0,K}(t) + \lambda P_{1,K-1}(t) \qquad (4)$$

$$\frac{d}{dt}P_{1,K-1}(t) = -(\lambda + \alpha + \propto)P_{1,K-1}(t) + \lambda P_{1,K-2}(t) + (1-\delta)\beta P_{2,K-1}(t) + \beta P_{3,K-1}(t) \qquad \ldots(5)$$

$$\frac{d}{dt}P_{1,n}(t) = -(\lambda + \alpha)P_{1,n}(t) + \beta P_{3,n}(t) + (1-\delta)\beta P_{2,n}(t) + \gamma P_{0,n}(t) + \lambda P_{1,n-1}(t)$$
$$1 \le n \le N-1 \qquad (6)$$

$$\frac{d}{dt}P_{1,N}(t) = -(\lambda + \alpha)P_{1,N}(t) + \lambda P_{1,N-1}(t) + (1-\delta)\beta P_{2,N}(t) + \propto P_{1,N+1}(t) + \beta P_{3,N}(t) + \gamma P_{0,N}(t) \qquad (7)$$

$$\frac{d}{dt}P_{1,n}(t) = -(\lambda + \alpha + \propto)P_{1,n}(t) + \lambda P_{1,n-1}(t) + (1-\delta)\beta P_{2,n}(t) + \propto P_{1,n+1}(t) + \beta P_{3,n}(t) + \gamma P_{0,n}(t)$$
$$N+1 \le n \le F \qquad (8)$$

$$\frac{d}{dt}P_{1,n}(t) = -(\lambda + \alpha + \propto)P_{1,n}(t) + \lambda P_{1,n-1}(t) + (1-\delta)\beta P_{2,n}(t) + \propto P_{1,n+1}(t) + \beta P_{3,n}(t)$$
$$F+1 \le n \le K-2 \qquad (9)$$

$$\frac{d}{dt}P_{2,0}(t) = -(\lambda + \beta)P_{2,0}(t) + \alpha P_{1,0}(t) \qquad (11)$$

$$\frac{d}{dt}P_{2,n}(t) = -(\lambda + \beta)P_{2,n}(t) + \alpha P_{1,n}(t) + \lambda P_{2,n-1}(t),$$
$$1 \le n \le K-2 \qquad \ldots(12)$$

$$\frac{d}{dt}P_{2,K-1}(t) = -\beta P_{2,K-1}(t) + \alpha P_{1,K-1}(t) + \lambda P_{2,K-2}(t) \qquad (13)$$

$$\frac{d}{dt}P_{3,0}(t) = -(\lambda + \beta)P_{3,0}(t) + \delta\beta P_{2,0}(t) \qquad (14)$$

$$\frac{d}{dt}P_{3,n}(t) = -(\lambda + \beta)P_{3,n}(t) + \delta\beta P_{2,n}(t) + \lambda P_{3,n-1}(t),$$
$$1 \le n \le K-2 \qquad (15)$$

$$\frac{d}{dt}P_{3,K-1}(t) = -\beta P_{3,K-1}(t) + \delta\beta P_{2,K-1}(t) + \lambda P_{3,K-2}(t) \qquad (16)$$

These equations have been solved using Runge-Kutta 4th order method and consequently the transient probabilities have been obtained. These probabilities have been used to determine the important performance measures related to the system in the subsequent section.

V PERFORMANCE MEASURES

For the model under consideration the following performance measures have been evaluated:

The queue length of the arriving WIP inventories is given by

$$E[N(t)] = \sum_{n=0}^{K} nP_{0,n}(t) + \sum_{n=0}^{K-1} nP_{1,n}(t) + \sum_{n=0}^{K-1} nP_{2,n}(t) + \sum_{n=0}^{K-1} nP_{3,n}(t) \qquad \ldots(17)$$

The throughput of the system is

$$TP(t) = \propto \sum_{n=N+1}^{K-1} P_{1,n}(t) + \sum_{n=1}^{K} P_{0,n}(t) \quad \ldots(18)$$

- The expected delay of the entire system is given by

$$ED(t) = \frac{E[N(t)]}{TP(t)} \quad \ldots(19)$$

- The probability of the server being busy is obtained as

$$P[B(t)] = \sum_{n=1}^{K} P_{0,n}(t) + \sum_{n=N}^{K-1} P_{1,n}(t) \quad (20)$$

- The probability of the server being under 1st phase repair

$$P[R_1(t)] = \sum_{n=0}^{K-1} P_{2,n}(t)$$
…(21)

- The probability of the server being under 2nd phase repair

$$P[R_2(t)] = \sum_{n=0}^{K-1} P_{3,n}(t) \quad \ldots(22)$$

- The probability of the server being idle is obtained as

$$P[I(t)] = P_{0,0}(t) + \sum_{n=0}^{N-1} P_{1,n}(t) \quad (23)$$

VI NUMERICAL ILLUSTRATION

The remanufacturing system modeled here has been illustrated numerically to study the effectiveness of various parameters on the system's performance. The effectiveness is displayed in tabular as well as in graphical form. Table 1 displays the effect of λ, α and t on the probability of the server being busy P[B(t)], probability of the server being idle P[I(t)], probability of the server being under 1st phase repair $P[R_1(t)]$ and the probability of the server being under 2nd phase repair $P[R_2(t)]$. It is observed that as t increases P[B(t)], $P[R_1(t)]$ and $P[R_2(t)]$ increase whereas P[I(t)] decreases. As the arrival rate λ increases, it results in the increment of P[B(t)] while P[I(t)] comes down and there is no change in $P[R_1(t)]$ and $P[R_2(t)]$. The increasing value of α shows a decreasing trend in P[B(t)] and P[I(t)] whereas there is an increment in $P[R_1(t)]$ and $P[R_2(t)]$. Table 2 depicts the effect of γ, α and t on P[B(t)], P[I(t)], $P[R_1(t)]$ and $P[R_2(t)]$. With the increasing values of γ and t, P[B(t)], $P[R_1(t)]$ and $P[R_2(t)]$ increase while P[I(t)] decreases. The effect of α on these probabilities is same as that observed in table 1. Fig. 2 (a), 2(b) and 2(c) display the behavior of queue length E[N(t)] for the increasing λ, α and β respectively w.r.t. the increasing time t. From fig 2(a), it is noticed that E[N(t)] rises sharply with both λ and t. Fig. 2(b) exhibit the queue length's increasing pattern with the increasing α and time t. the increment is significant with the increasing time whereas the increment is not much for the increasing values of α. From fig. 2(c), a remarkable rise in the queue length is evident with the increasing time t but as the β is increased, the queue size reduces.

The impact of λ, α and β on the throughput TP(t) is portrayed in fig. 3(a), 3(b) and 3(c) respectively with the increasing time t. It is observed that TP(t) rises with both λ and t; the increment is much for higher values of λ. Fig. 3(b) displays an increasing trend of TP(t) with the rising time however, it comes down with the increasing α. In fig 3(c) we observe that the TP(t) rises slowly with the increasing time t, in addition a small increasing variation is noticed with the increasing values of β

VII CONCLUSION

In this paper an unreliable remanufacturing system for the WIP inventory is studied. The system provides the service to the arriving units according to N-policy, F-policy or both the policies simultaneously. Thus, the inclusion of the concepts of (N, F) policy and unreliability of the server makes the model more realistic. By performing the mathematical modeling of such a remanufacturing system by incorporating the above realistic concepts, the system designer may obtain favorable results beneficial to the concerned organization. On the basis of the results obtained from the numerical experiment, some important decisions can be made so that the queue size can be reduced and the throughput can be improved.

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