Inversion of Integral Equation Involving Polynomial Suggested by Hermite Polynomial

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ABSTRACT
The purpose of this paper is to drive a solution of certain integral equation whose kernel involves Generalized Hermite Polynomial. We believe that our result is unify in nature and many results can be obtained by considering suitable parameters involved in Generalized Hermite Polynomial. For the purpose of illustration we mentioned a special case briefly by choosing suitable parameters involved in Generalized Hermite Polynomial.

Keyword: Generalized Hermite Polynomial, Mellin Transform, Convolution Theorem, Fox-H function.

I INTRODUCTION

Many boundary value problems reduced to the problem of solving integral equations whose kernel involves many well known classical polynomials like those of Hermite, Laguerre, Bessal, Legendre, Jacobi etc. During the recent past attempts have been made to generalize and unify these classical polynomials with the help of Rodrigue's formulae. To mention Goued Hopper [8] gave a generalization of Hermite polynomials by formulae.

$$H_n^r\left(x,a,p\right)=(-1)^n x^{-a} e^{px^r} D^n\left[x^a e^{-px^r}\right]$$

(1.1)

and we have used

$$H_n^2\left(x,0,1\right)=(-1)^n e^{x^2} D^n\left[e^{-x^2}\right]$$

(1.2)

where $D\equiv\frac{d}{dx}$ and $r, a,$ and p are parameters, for suitable value of $r, a,$ and p (1.1) reduced to modified Hermite, modified Laguerre and modified Bessel polynomials. In view of these generalizations it is worth considering integral equations involving $H_n^2\left(x,0,1\right)$ as kernel and such we prove the following theorem.

II THEOREM

If f is an unknown function satisfying the integral equation.

$$g\left(x\right)=\int_0^\infty k\left(x\mid y\right) f(y)\frac{dy}{y}, x>0$$

(2.1)

Where $k(x)=e^{-x^2}.H_n^2\left(x,0,1\right)$

and g is a prescribed function then f is given by

For $r=2, p=1$

$$f(x)=(-1)^n x^n$$

$$\times\int_0^\infty 2H_{3,2}^{2,0}\left[\frac{x}{y}\Bigg|\begin{matrix}(0,1)(-n,1/2)(-n,1)\\(-2n,1)(-n,1)\end{matrix}\right]$$

$$\times\left[\left(\frac{d}{dy}\right)^n\left\{g\left(y\right)\right\}\right].\frac{dy}{y}$$

III SOLUTION

To Prove the Theorem we make use of Mellin Transform and discuss case $r>0$, $p>0$.

$(r=2, p=1 \text{ in our case})$

By the convolution theorem for Mellin Transform (2.1) reduces to

$$k^*(s)f^*(s)=g^*(s)$$

(3.1)

Where $k^*\left(s\right), f^*\left(s\right), g^*\left(s\right)$ are respective Mellin Transform of $k\left(x\right), f\left(x\right), g\left(x\right)$ and by Sneddon

$$f^*(s)=\int_0^\infty f(x)x^{s-1}dx=M\left[f(x),s\right] \qquad (3.2)$$

When $r>0$ and $p>0$, Applying Mellin Transform of equation (3.1) and use the result of Erdelyi $k^*(s)=(-1)^n M\left[D^n\left(e^{-x^2}\right);s\right]$ [9], we get

$$=\frac{\Gamma(s)\,\Gamma\left(\frac{(s-n)}{2}\right)}{2\,\Gamma(s-n)} \qquad (3.3)$$

Where

$\mathrm{Re}(s) > n, \quad where \quad \mathrm{Re}(a) = 0$

$\mathrm{Re}(s) > n - \mathrm{Re}(a), \quad where \quad \mathrm{Re}(a) \le 0$

We write equation (3.1) in the form

$$f^{*}(s) = \frac{g^{*}(s)}{k^{*}(s)}$$

replacing s by $s - n + a$ where $a = 0$

$$f^{*}\left(s-n\right) = (-1)^{n} L^{*}(s)\left[\left(-1\right)^{n} \frac{\left|\overline{s}\right.}{\left|\overline{s-n}\right.} .g^{*}\left(s-n\right)\right]$$

(3.4)

Where

$$L^{*}(s) = \frac{\left|\overline{s-n}\right.}{\left|\overline{s}\right. .k^{*}\left(s-n\right)}$$

(3.5)

Then from (3.3) and (3.5)

$$L^{*}(s) = \frac{2\left|\overline{s-2n}\right.\left|\overline{s-n}\right.}{\left|\overline{s}\right.\left|\overline{s-n}\right.\left|\overline{\frac{s-2n}{2}}\right.} \qquad (3.6)$$

By use of definition of H function. We get the inverse transform $L(x)$ of $L^{*}(s)$ as

$$L(x) = 2H_{3,2}^{2,0}\left[x \,\Big|^{(0,1)(-n,1/2)(-n,1)}_{(-2n,1)(-n,1)} \right] \qquad (3.7)$$

Where $H_{m,n}^{p,q}$ are Fox's H functions defined by [5].

And now taking Mellin Transform on both sides of (3.4), using convolution theorem and result of Mellin Transform. We get

$$M^{-1}\left[f^{*}(s-n)\right]$$

$$= (-1)^{n}\int_{0}^{\infty} L\left(\frac{x}{y}\right)\left[M^{-1}\left\{(-1)^{n}\frac{\left|\overline{s}\right.}{\left|\overline{s-n}\right.}g^{*}(s-n)\right\}\right]\frac{dy}{y}$$

$$x^{-n} f(x) = (-1)^{n}\int_{0}^{\infty} L\left(\frac{x}{y}\right)\left[\left(\frac{d}{dy}\right)^{n}\left\{g(y)\right\}\right]\frac{dy}{y}$$

$$f(x) = (-1)^{n} x^{n}\int_{0}^{\infty} L\left(\frac{x}{y}\right)\left[\left(\frac{d}{dy}\right)^{n}\left\{g(y)\right\}\right]\frac{dy}{y}$$

Hence using (3.7)

$$f(x) = (-1)^{n} x^{n}$$

$$\times \int_{0}^{\infty} 2H_{3,2}^{2,0}\left[\frac{x}{y}\Big|^{(0,1)(-n,1/2)(-n,1)}_{(-2n,1)(-n,1)} \right]$$

$$\times\left[\left(\frac{d}{dy}\right)^{n}\left\{g\left(y\right)\right\}\right].\frac{dy}{y} \qquad (3.8)$$

Thus we have prove the following theorem – If f is unknown function satisfying (2.1), where g is some known function then f is given by (3.8) according $r > 0$.

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